Nurturing the talent of gifted children ages 12 to 15: A project of Miguel de Guzmán

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The official retirement day for Miguel de Guzmán would have come on September 30, 2006, only one month after the close of the International Congress of Mathematicians in Madrid. Many have contributed to making such an event possible for the first time in Spain, but one of the first steps in this direction was taken by Miguel de Guzmán, who was born in 1936 and died in 2004, and who nurtured a passion for mathematical research that has prevailed in Spain for generations. Near the end of this Congress no tribute has been paid to a man who has made a remarkable contribution to mathematics in Spain, both at the research and educational level. **This presentation is dedicated to Miguel.**

We are not here to talk about his many contributions to research but rather to present one of the programs he started in mathematics education. As far back as 1995, in a visit that my family and I paid to Miguel and his wife Mayte, he told me about the idea of developing a program for talented young students. It is better to use his description of the program:

Without doubt, there are in our school communities a number of students endowed with a truly exceptional intellectual gift for mathematics. These are talents that at times will go more or less unnoticed and somewhat ignored, since it is impossible for their teachers to devote to these pupils the personal attention they require.

Their talent, moreover, if not wasted, could bear remarkable fruit for the common good of society through their extraordinary contributions to the country's cultural, scientific and technological development. We as a society bear tremendous responsibility for the unquestionable waste of talent caused by neglecting these intellectual gifts.

It would be unfair to say that this is a unique program in the world. There have been many countries in which a great deal of attention has been paid to this issue. Among the many possible ways of dealing with it, our program is similar to those initiated first at Johns Hopkins University in Baltimore, in the US, and later followed by the city of Hamburg, Germany. A presentation of both programs took place in the Facultad de Matemáticas of the UCM in November 1998.

The program's **objective is to detect, stimulate and guide** the talent of mathematically gifted children, aged twelve to fifteen, without removing them from their school environment.

(The first letters of the Spanish words EStimulo, TALento and MATemáticas gave rise to the name ESTALMAT, as the program is now known.)

This stimulation and guidance is done continuously, i.e. not merely by specific events such as competitions, but by means of a steady line of activities. The method chosen consists of three-hour meetings once a week during the academic year. The age group (twelve–thirteen, when they start) has been considered the most appropriate for different reasons. It is more or less the time of the awakening of formal reasoning. In addition, experience in other countries that have guided us involved work with children of this age.

HISTORY OF ESTALMAT

Before we start with the description of the program, we must describe, briefly, the (not very long) history of ESTALMAT and pay due tribute to our sponsor.

In 1998, the Spanish Royal Academy of Sciences decided to start this project. For practical reasons, the Academy started out with a pilot program in the region of Madrid. In 2000, the former president of the Academy, Angel Martín Municio, reached an agreement with the Airtel Foundation, which funded the program for the next two years.

When the mobile telephone company Vodafone bought Airtel, the Vodafone Spain Foundation continued funding the program. Seven groups have already graduated, if you will, in Madrid, and selection of the ninth generation Madrid participants took place in June 2006.

The financial contribution of the Vodafone Spain Foundation allowed the program to be extended to other regions of Spain. In 2003, the program started in Catalonia and in the province of Burgos. In 2005, Estalmat started in western Andalusia and the Canary Islands; at the same time, the Burgos branch of the program was extended to other provinces of Castille: Valladolid, León, Segovia and Zamora. In September of 2006 Estalmat will start in eastern Andalusia.

THE SELECTION PROCESS

We consider the selection process to be of prime importance. It is therefore a task that engages us for extensive time during the winter and spring of each year. These are the steps that are taken:

First stage:

- The project is announced and publicized among students, teachers and parents during the spring.
- Students that participate in local mathematics competitions are a good target for this promotion. Since different regions of Spain have different local competitions, information about the project is distributed among students 12-13 years of age during these events.
- The math teachers working in all primary and secondary schools of the region are informed of the project by letter or electronic announcements to their schools asking

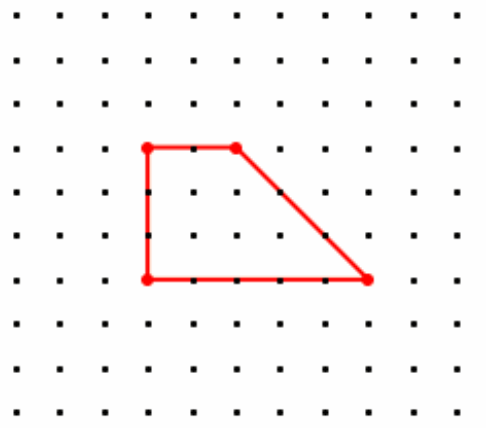
them to encourage their best students to take part in the selection process. We believe that teachers of elementary and secondary schools must play an important role in identifying gifted students.
- Parents are also brought into the process through print ads in regional newspapers. Web pages for each region are also available.
- To participate in the selection process, a student must be recommended by his or her math teacher and entered by his or her parents, which is usually done by mail to the registration office of each project with the required information. Online registration is available in some regions.

Second stage:
- Students recommended by teachers and parents must show their mathematical skills by working on several problems posed to them. One of the most important aspects to keep in mind is that the goal is not to assess their mathematical knowledge, but their mathematical potential.
- Once their mathematical skills are evaluated and the first-round candidates have been chosen, separate interviews are held with the finalists and their parents to evaluate their enthusiasm for participating in the program and the willingness of the parents to make the sacrifices it requires. Although the project involves no cost to the children's families, parents do have to be willing to commit to making the effort to chauffeur their children to the three-hour activities over the course of two years.
- Twenty-five students, or fewer in the case of small cities, are chosen as participants.

The problems:
- The candidates are given from four to six problems (depending on the region) to work on. Since our intention is to make sure that these are original problems to make it impossible for students to prepare for the exam, we are constantly on the lookout for problems throughout the year, and we ask ESTALMAT teachers to provide suggestions for problems to be included.
- The problems the candidates are asked to solve must be written in clear language. It is important to ensure that the difficulty does not lie in the language used to describe the problem.
- The problems are designed to deal with several aspects of reasoning: visual thinking, logical reasoning, intuition, creativity, geometrical skills, abstraction, and mathematical manipulation.
- The questions posed in each problem move progressively from less to more difficult. The idea is for all of the candidates to be capable of answering at least part of the problem, but only those showing mathematical talent will be able to answer the higher level questions.

Here is an example from the selection process of 2005.

In this problem we consider very special trapezoids. They must have two right angles and a 45º angle and all its vertices must be points of a square grid. Look at the trapezoid in the figure: Since it contains 18 points, counting those that are on its sides, we say that 18 is a **trapezoid number**.



a) Draw a figure to show that 35 is a trapezoid number.
b) Draw all trapezoids having 18 as a trapezoid number and justify that these are all you can draw.
c) Explain why any odd number greater than 3 is a trapezoid number.
d) Find all numbers between 4 and 50 that are not trapezoid numbers.

Number of candidates

In previous presentations of ESTALMAT one of the questions raised by the audience was the number of candidates that enter the selection process. Here are some approximate numbers:

Madrid -- population: 5.8 million --
Candidates in 2005: 250
Candidates in 2006: 235

Catalonia -- population:
Candidates in 2006: 257

Western Andalusia -- population:
Candidates in 2005: 1090
Candidates In 2006: 673
Eastern Andalusia -- population:
Candidates in 2006: 588

Castille and León (Burgos, León, Segovia, Valladolid and Zamora) – population:
Candidates in 2006: 538

PROGRAM ACTIVITIES

Once the students are selected in June, they start the program in September with a two-day weekend meeting, usually in a rural setting. The meeting allows them to get to know both each other and some of the teachers involved in the program. The kids participate in both mathematical and outdoor activities over the two-day weekend. We take this opportunity to observe any special interaction among the students and use it to

divide them into groups for the weekly sections. This is an important part in the success of the activities and it must be done carefully.

The main activity of the project with these children lasts for two years. Once a week the students participate in three hours of activities totally removed from the content of their syllabus at school. The three hours of activities are divided into two sections of 1 hour and 20 minutes each, with a break of about 20 minutes between them. Students are divided into small groups (usually 5 per group) and under the guidance of two teachers, some of them university professors and others secondary school teachers, they explore subjects such as the following:

- **Graphs , Prime Numbers, Game Strategies, Polyhedra, Interactive Geometry with the computer, Mosaics, Counting, Divisibility, Parity, the Pigeon Hole Principle, Ramsey Theory, Fractals….**

The main goal of these activities is to create the appropriate conditions to develop the students' mathematical creativity. Whether the sessions are dedicated to explaining advanced mathematical topics or to discovering relevant results in mathematics, there is always an abundance of challenging problems to work on. In preparing these activities we used our own expertise and a wide range of literature, of which special mention must be given to the book "Mathematical Circles (Russian Experience)" by Dmitri Fomin, Sergey Genkin and Ilia Itenberg, published in English by the AMS.

It would be impossible to do a complete description of all the activities carried out during this period. But a few words must be said about them. A sample of these activities (2 from each region) have been collected in a CD, which can be picked up at the end of this presentation. (The information will also be available on the Estalmat Web page of the Madrid project.)

When presenting well established mathematical subjects to these students (i.e., graphs, probability) a short introduction is made by the teacher and activities are proposed to guide them through the main concepts of the subject and to encourage them to discover results. During this part, the students work in groups, proposing ideas and solutions that must be discussed with their teammates. These ideas and solutions are finally collected and developed under the guidance of the teacher, who comments about them and ask the students to defend their points of view.

On other occasions the teacher starts by presenting a problem, which in principle could be quite complex, and asks the students to provide solutions in particular cases or to simplify the problem to try to seek a solution. As an example, consider overbooking by airlines.

Overbooking

Why do airlines usually sell more tickets than seats available on the plane? Think from the point of view of the airline. Suppose that the data concerning a flight are as follows:

- ♣ The plane has N=100 seats.
- ♣ Price of each ticket is P=120 €
- ♣ If a passenger cannot fly due to overbooking, he/she has the right to a refund for the whole price of the ticket, and a compensation of 250 €
- ♣ The fixed cost of flying an airplane is C=5000 €

Experience tells us that a certain number of passengers do not show up for the flight. Consider the following two options:

- To sell exactly as many tickets as seats are available on the plane. What is the benefit of the company in this case?
- To sell n tickets, where n > N. Choose n in order to maximize benefits.

Probabilistic model

- ♣ Consider as a first approximation that the probability of a passenger not showing up for the flight is p (say 10%) and is the same for all passengers.
- ♣ Suppose also that the decision of each passenger to fly does not depend on other passengers' decisions.

The purpose of this activity is to answer the following question: **With the above considerations, what is the value of n that we must choose to maximize profits?**

Variations

We want to make the probabilistic model more realistic to take into account the following situations**:**

- There are two types of customers: business and economy.
  - ♣ We know that 20% of business passengers do not show up for the flight.
  - ♣ 5% of economy passengers do not show up.
  - ♣ Also, 60% of the passengers on this flight are business class. Moreover, some customers do not fly alone. For example, a percentage of them are families that fly together.

Can you use the above model to analyze these situations? What would you change?

Sometimes the project proposed is completed, but most often it is not, leaving questions unanswered for interested students to pursue. At this point it is important to mention that there are no homework assignments for the students between sessions, since we don't want the students to be distracted from their school studies at school, where other important subjects for their development are explored. (This was a point on which Miguel de Guzmán was very adamant.) Of course, thinking about unanswered questions is not prohibited, and they can turn in written ideas and discuss with the teachers their solutions to these questions.

It is also important to emphasize that this program is not aimed at preparing students for mathematics competitions. I don't want to give this audience the impression that we have anything against math competitions. On the contrary, these competitions, at all levels, are an important part of the development of science in society, and many Estalmat teachers should be credited for developing an extraordinary system of competitions in many regions of Spain. In any case, some Estalmat students, of their own accord, do enter competitions and some have reached levels in the International Mathematical Olympiad that have never been achieved before by Spanish representatives.

At this point I would like to read to you a couple of comments by one of the four Fields Medallists (or should I say three?), Terence Tao, in an interview held after the opening ceremony of this Congress (you can read the whole interview on the ICM2006 Website).

In my opinion the most important thing for developing an interest in mathematics is to have the ability and the freedom to "play" with them.

……

To learn theory and applications, and to see a subject as a whole, there is no doubt that the formal environment of the classroom is the best. But the classroom is not the best place to learn to experiment.

(Terence Tao, August 22, 2006)

With the Estalmat activities we try to provide a place to experiment and the freedom to play with concepts and to discuss ideas that blossom from the ability of the students selected to ask interesting questions.

After the first two years of intensive dedication to the project, we offer interested students the option of returning to the project once a month (at least in Madrid where the project has been run for 8 years and 7 "generations" have already graduated from Estalmat) and exploring deeper and more sophisticated mathematical activities. This allows us to keep in touch with the previous Estalmat students until they reach the University level.

PECULIARITIES OF EACH REGION

Although we share the same objective, each region has its own peculiarities inside Estalmat. We must say a few words about them.

The program started in Madrid in 1998, funded first by the Spanish Royal Academy and since the year 2000 by the Vodafone Spain Foundation. The 25 students selected each year meet at the Mathematical Sciences building of the Universidad Complutense de Madrid, which has generously offered its facilities to this project from the very beginning.

The program in Catalonia started in 2003. Also selecting 25 students, it extends to the four provinces of the Catalonia region, and is held on the premises of the Polytechnical University of Catalonia, in Barcelona. Funded completely by the Vodafone Spain Foundation, the project is run under the auspices of FEEMCAT (Federació d'Entitats per a l'ensenyament de les Matemàtiques de Catalunya) and SCM (Societat Catalana de Matemàtiques).

In Castille and Leon the program is run under the auspices of the Sociedad Castellano y Leonesa "Miguel de Guzmán" with five venues in Burgos, Valladolid, León, Segovia and Zamora. Funding is provided by the Vodafone Spain Foundation, the Foundation of the University of Burgos and local government.

In Andalusia, activities are held in Seville and Granada. The program is run in collaboration with the Sociedad Andaluza de Eduación Matemática THALES and all Andalusian universities. Funding is provided by the Vodafone Spain Foundation and by the San Fernando Bank.

Finally, The Regional Government of the Canary Islands started a program that joined Estalmat in 2005. The program is run in collaboration with the Sociedad Isaac Newton de Profesores de Matemáticas.

One of the dreams of Miguel was to extend the program as much as possible throughout Spain; some steps have been taken in this direction, but we still have a long

way to go. I would like to encourage all those present here today to try to set up a similar program in your region, whether in Spain, in Europe or in other parts of the world.

RESULTS

The evaluation made by the students at the end of each academic year shows a high degree of satisfaction from almost all participants: they prefer to work in groups rather than alone, and a high percentage has fun during the activities, although the majority thinks that the activities proposed are difficult. In this respect, the short-range goals of the program have been met completely.

The long-range success of the project will be determined if the participants make "extraordinary contributions to the country's cultural, scientific and technological development." But it is still too early to assess the long-term impact.

Some tendency can be shown from three generations of Estalmat students that have reached the university level (all of them from the Madrid branch). Of 69 students that have participated in Estalmat and have reached university age, 10 study Mathematics (6 only Math and 4 are majoring in Math and Computer Science at the same time); also 10 of them study Computer Science (5 of them Computer Science alone, while 4, as mentioned above, major in Math and Computer Science and 1 in CS and Business Administration); 9 study Industrial Engineering; 8 Telecommunications Engineering….. Of course, Engineering Schools get the most, with 36 of the 69.

We can say that what the project has already truly fulfilled in its current state is quite obvious. It has afforded these children a deep vision of what working with mathematics means, a vision that gives them great satisfaction. And this was the one of the goals Miguel de Guzmán and the Spanish Royal Academy of Sciences set out to achieve with ESTALMAT.

Some comments from students are a testimony to this fact:

- "This is a program for which, to tell the truth, I did not expect to be chosen. We learned very interesting things every Saturday, and we have not only learned math, but we have each made 24 friends, people who are both different from and similar to ourselves."
- "These are classes in which you learn many things, where there is a very good atmosphere, and where teachers help us a lot and teach us many things that without this opportunity I would not have learned."
- "I am going to classes held at the University where we do activities and games that use logic and mathematics. We also do proofs. I have met friendly people."

If you want to know more details about the selection process or the activities conducted, I invite you to visit the Poster session on Tuesday the 29th, from 17 to 18 PM on the third floor of this building, where one of the teachers from the program will be

available to answer your questions. Alternatively, you can visit our main Web page www.estalmat.org, where links to the websites of each Estalmat region can be found.

We finish by showing the poster of the Conference held in honor of Miguel on the 13th, 14th and 15th of December, 2004, and reading one of his quotes:

ESTALMAT is an encouraging experience, a gateway to a world of games and mathematical activities with a deep educational value. Of great interest to society and a delightful experience for all of us who participate in it.