

**Barcelona**

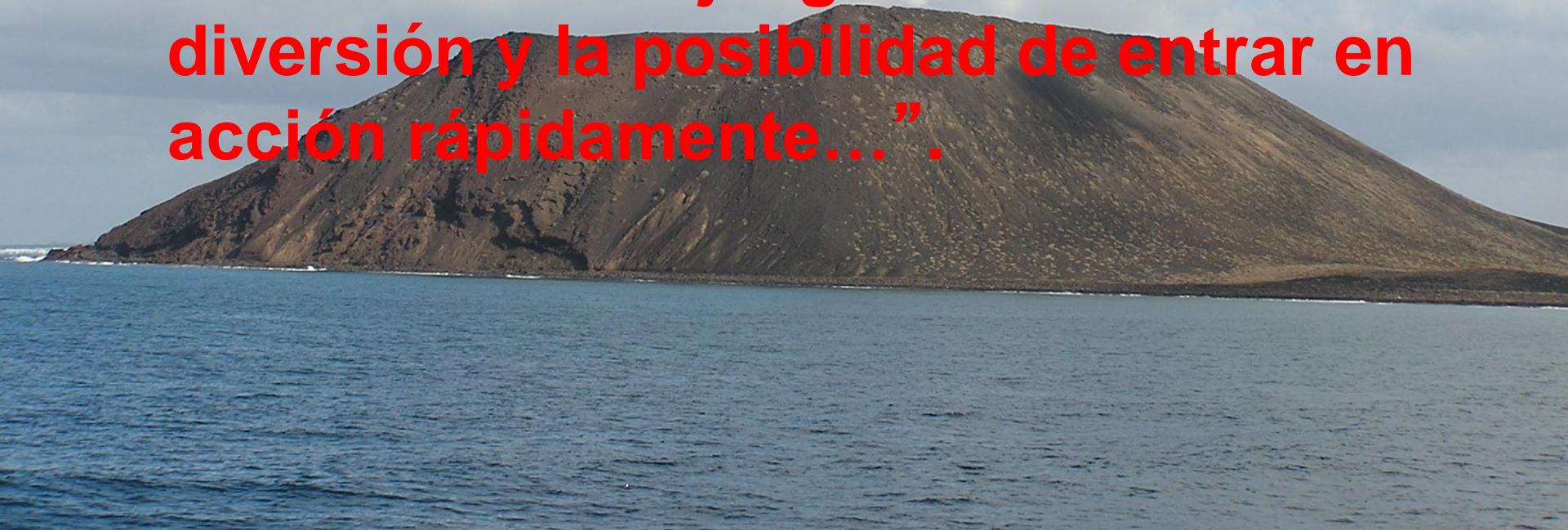
**Barcelona**  
**4,5,6 de Abril**

**VII SEMINARIO NACIONAL ESTALMAT**

# PROBABILIDAD, DIAGRAMAS DE ÁRBOL

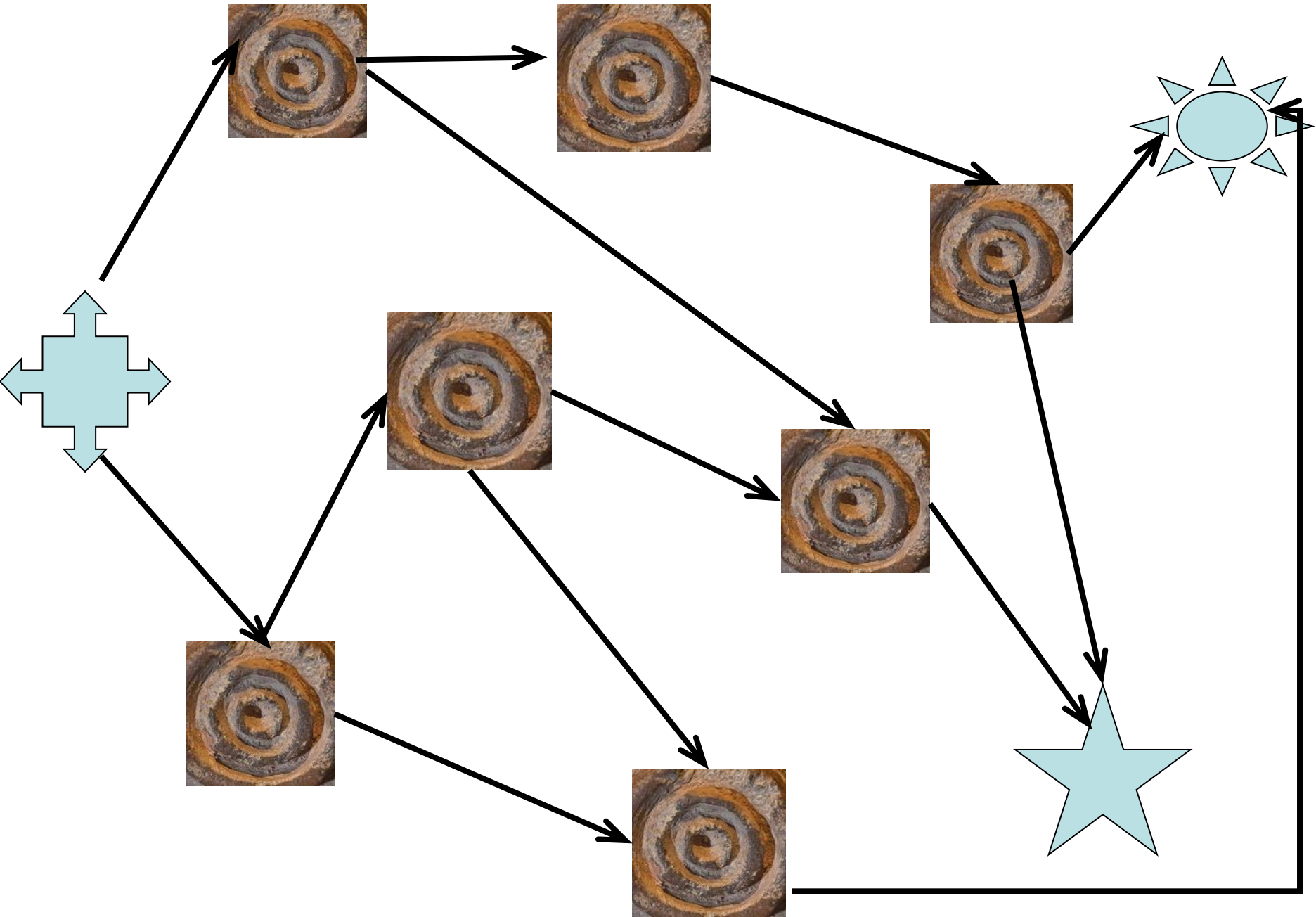
Luis López García  
Estalmat Canarias

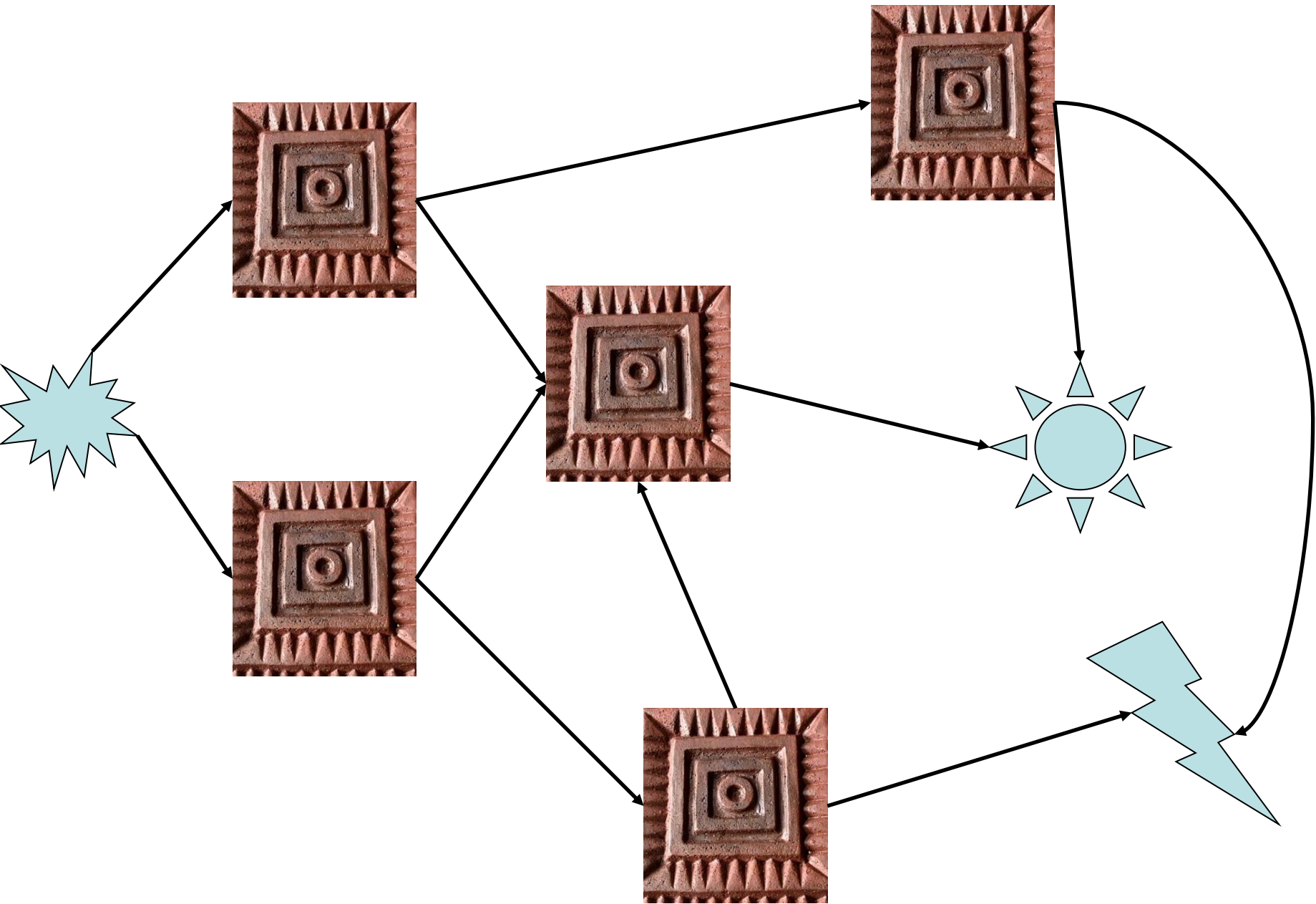
- **El propósito de este trabajo es presentar actividades que permitan la introducción a la probabilidad mediante una serie de juegos, teniendo en cuenta, como lo manifiesta Miguel de Guzmán:” en el juego se busca la diversión y la posibilidad de entrar en acción rápidamente...”.**

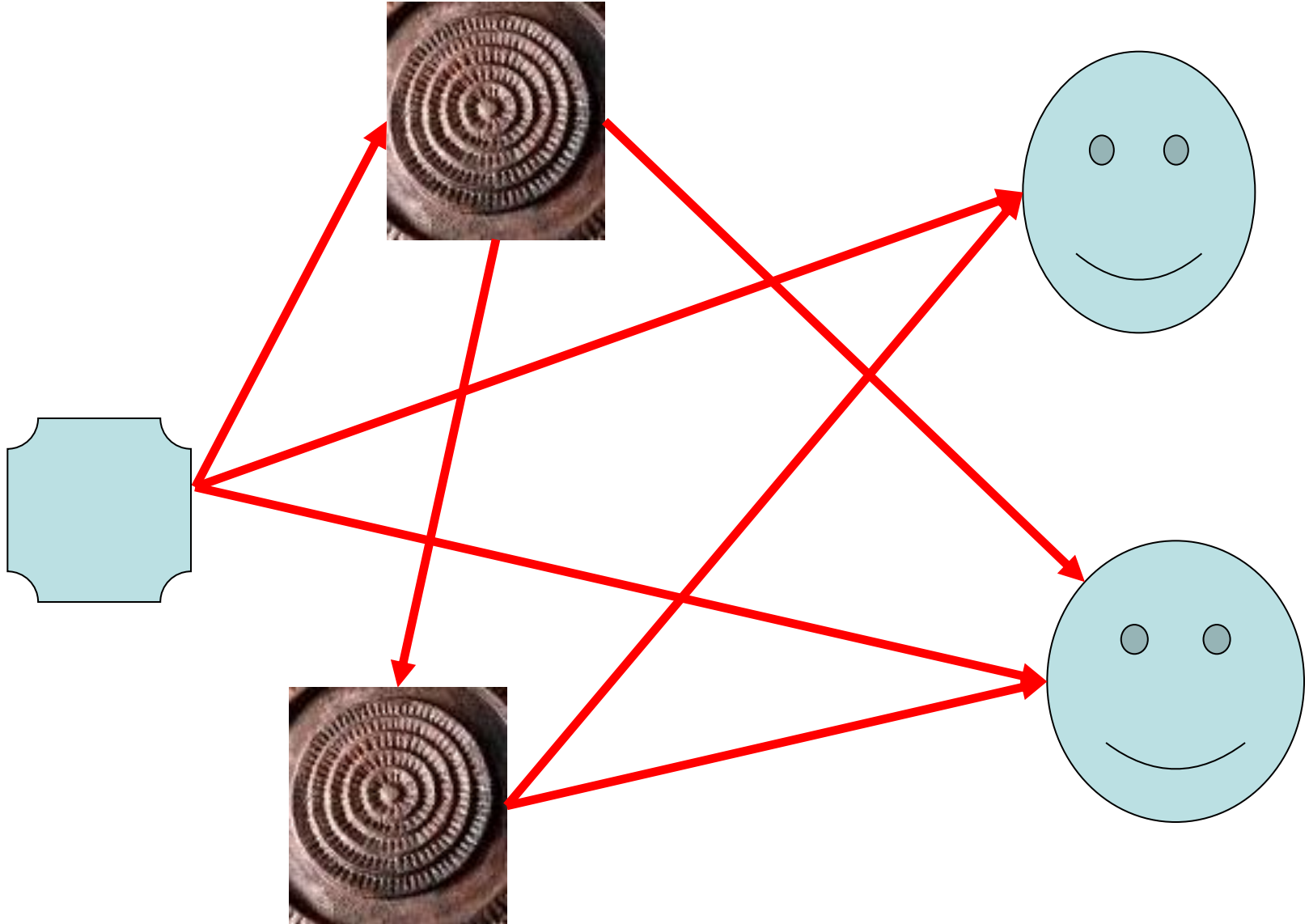


*Inicialmente a cada alumno se le entregan los tableros, una ficha, un dado, una peonza y una moneda.*









**Se coloca la ficha en la  
entrada del juego.**




*Se le dice que en cada camino se debe sortear el recorrido que tome la ficha en el siguiente movimiento. El sorteo lo debe realizar cada alumno.*



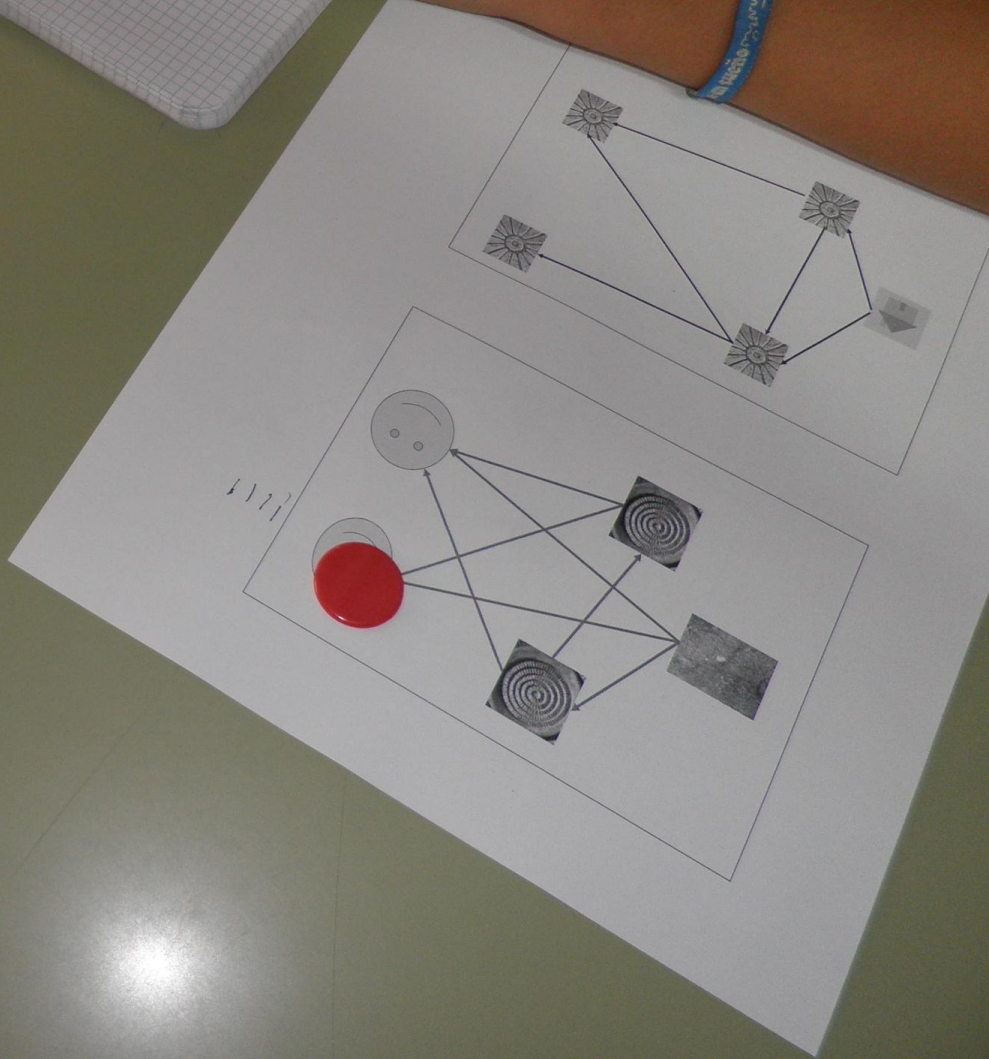
*Se les pide que jueguen un rato y que vayan anotando los resultados que vayan obteniendo.*



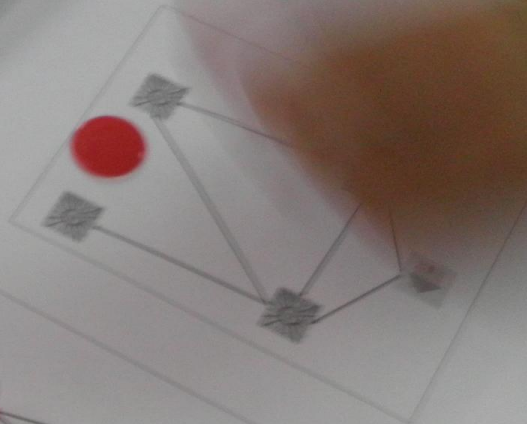
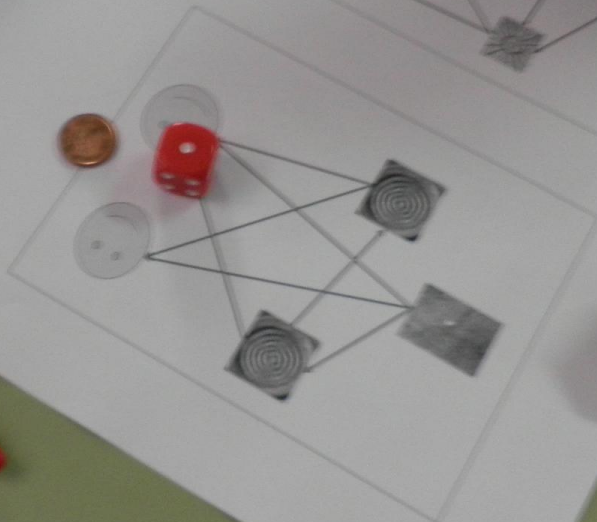
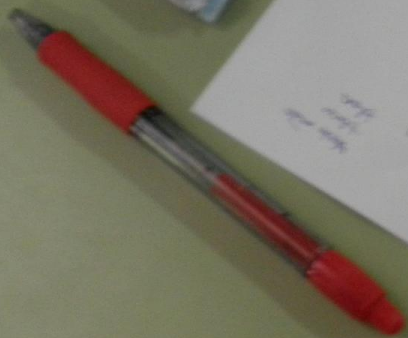
A black and white photograph of a volcanic landscape. In the foreground, a dark, sandy path leads through a field of dark volcanic ash and small, sparse green plants. The path curves towards the right. In the background, a low ridge of dark rocks and ash stretches across the horizon. The sky is a clear, bright blue. A white rectangular box with a thin black border is superimposed over the center of the image, containing text in a bold, italicized, black font.

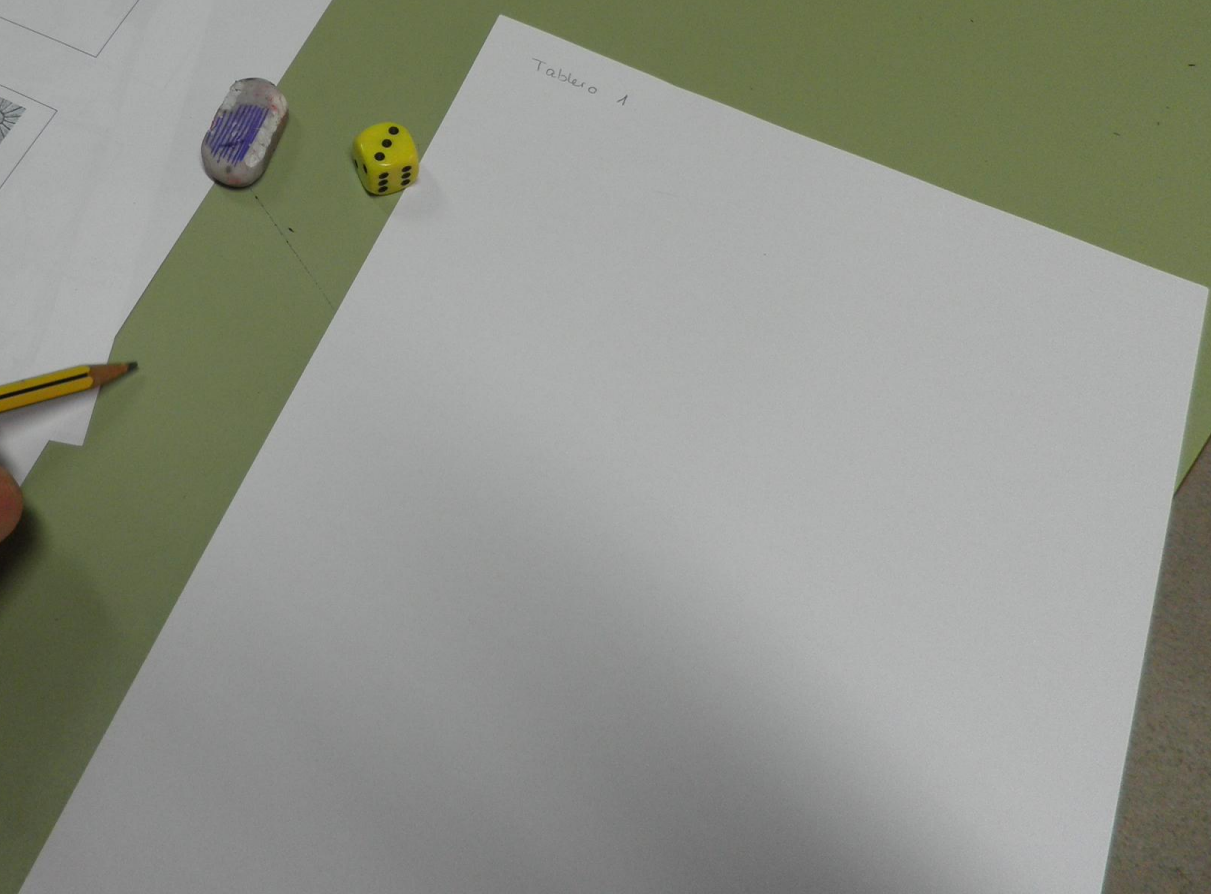
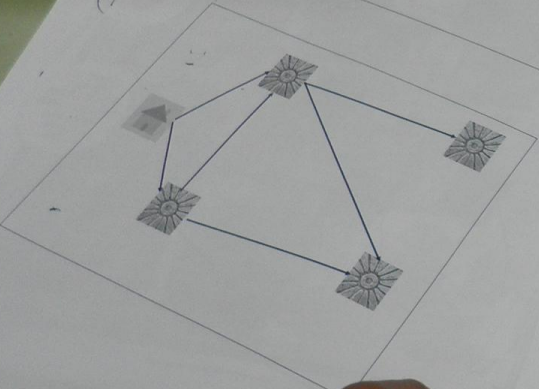
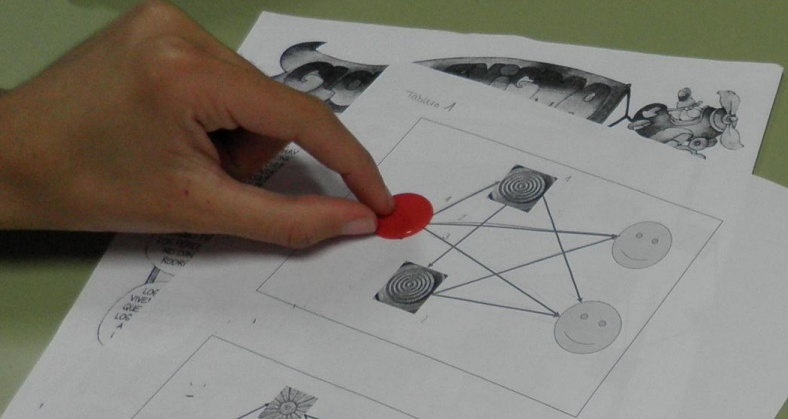
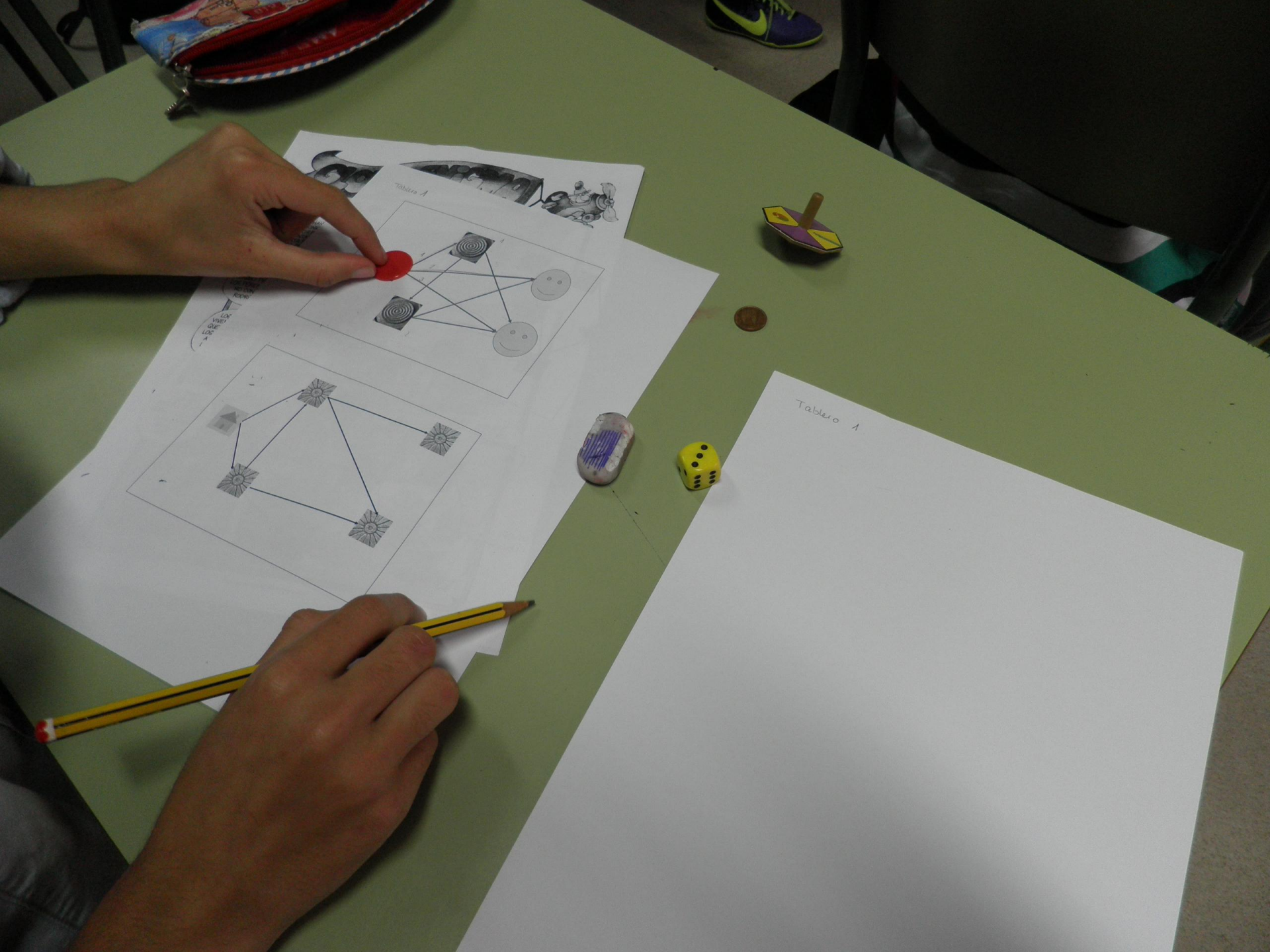
***Luego se les pide que expongan cómo y con qué han realizado el sorteo y los resultados obtenidos.***

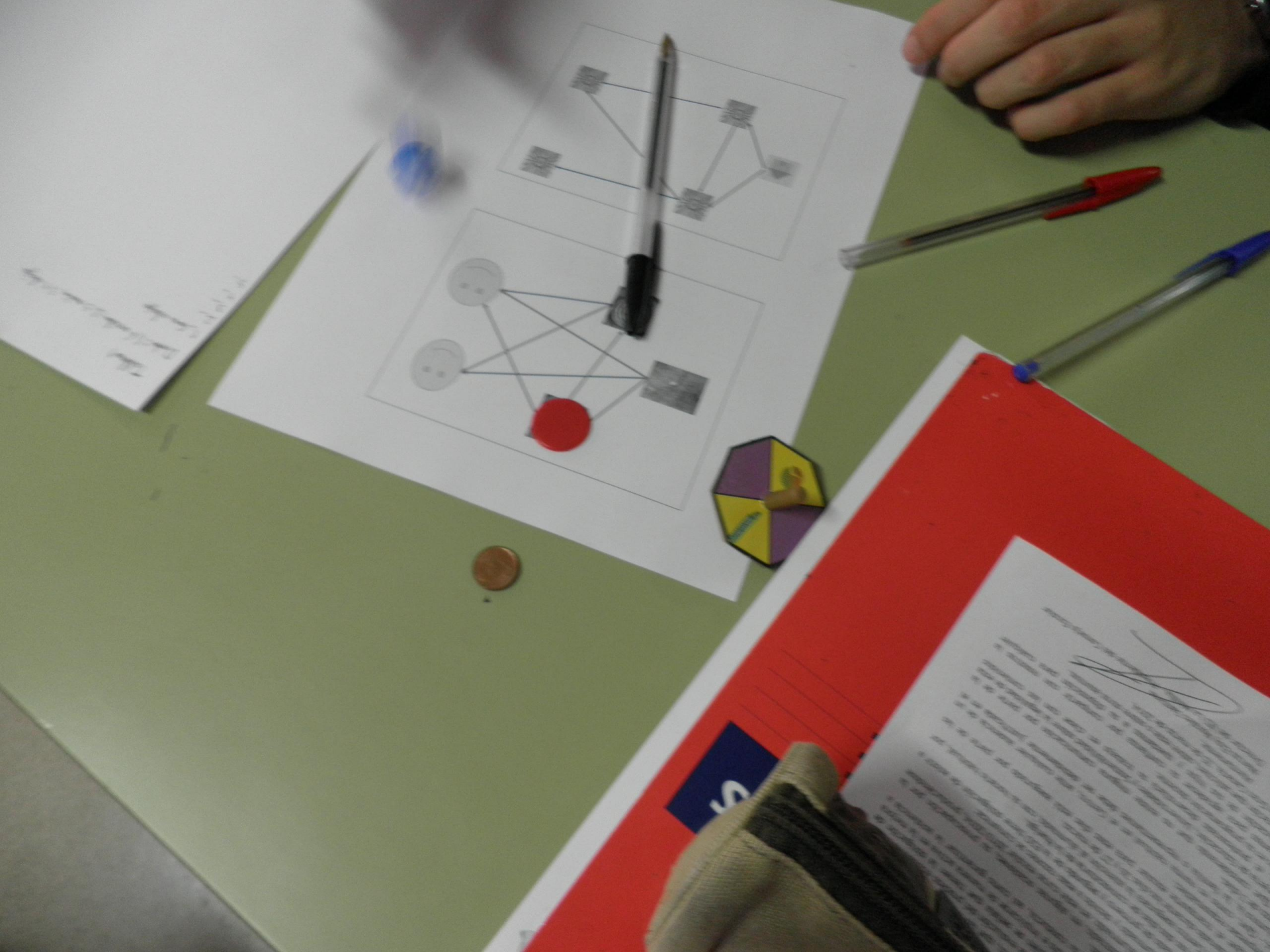
$\text{Residuals} = \text{Observed} - \text{Expected}$   
 $\text{Chi-squared} = \sum \frac{\text{Residuals}^2}{\text{Expected}}$



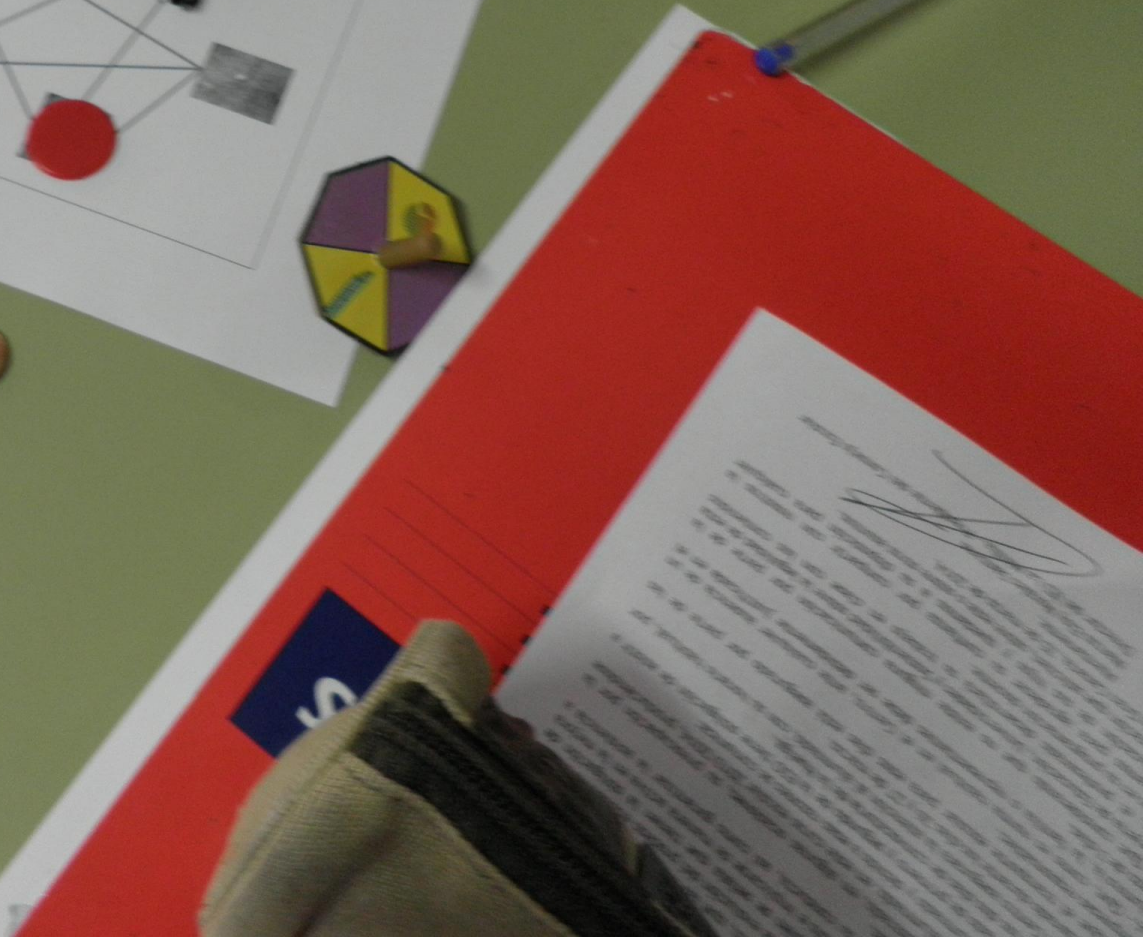
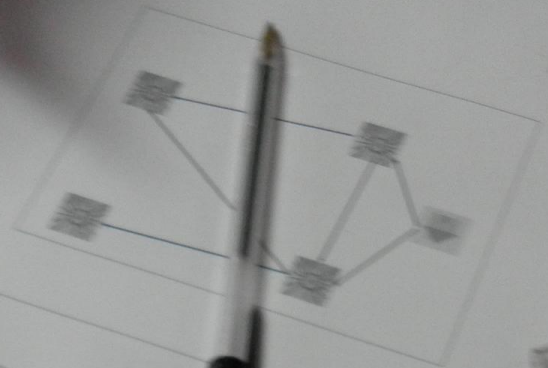
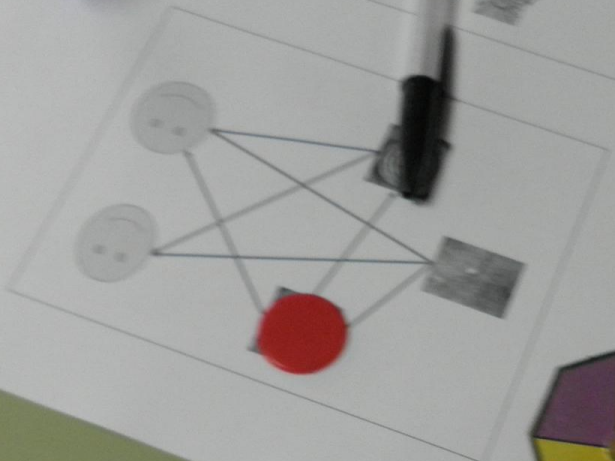
Handwritten notes on a light blue sticky note, partially obscured by a pen and other objects. The text is mostly illegible but appears to contain a list or set of instructions.







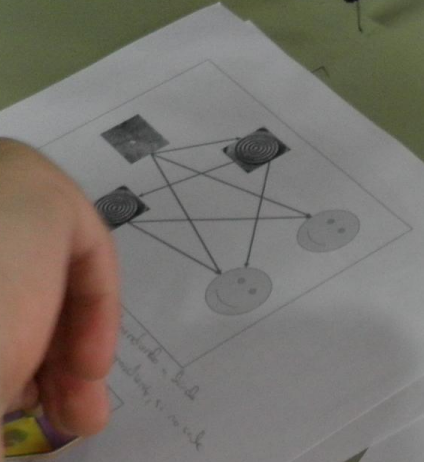
*[Faint handwritten notes on a white paper strip]*



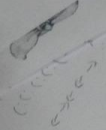








- Verme  
 - En el humor  
 - Aprendiendo de a que dirección a parte de la vida es que me  
 Si la vida apunta para abajo, entonces sea  
 solo de una en orden, bien y lo tomamos



- 1) hucha
- 2) 3
- 3) 4
- 4) 5
- 5) 6

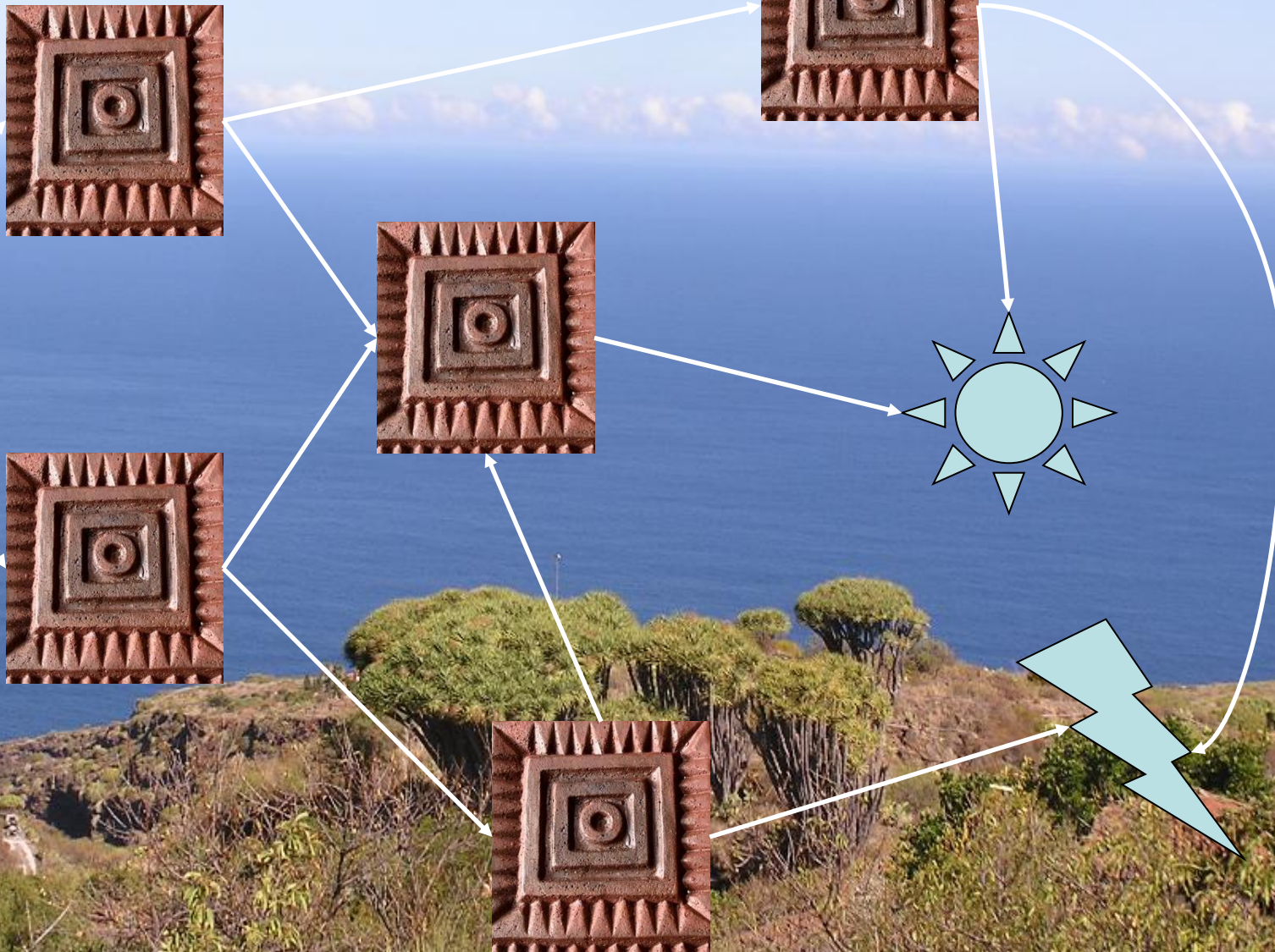
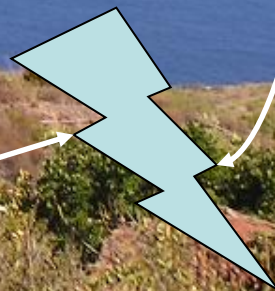
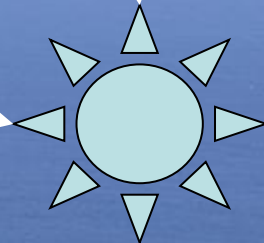
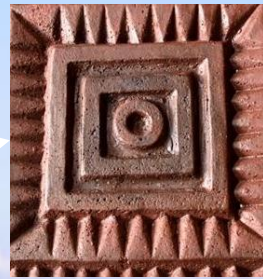
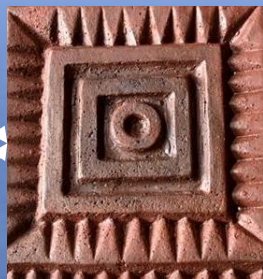
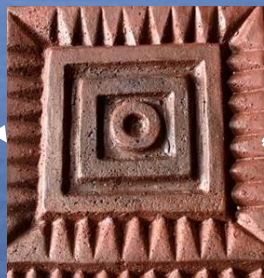
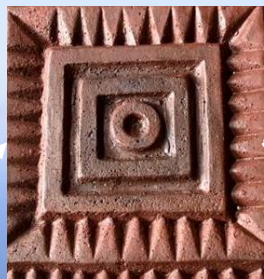
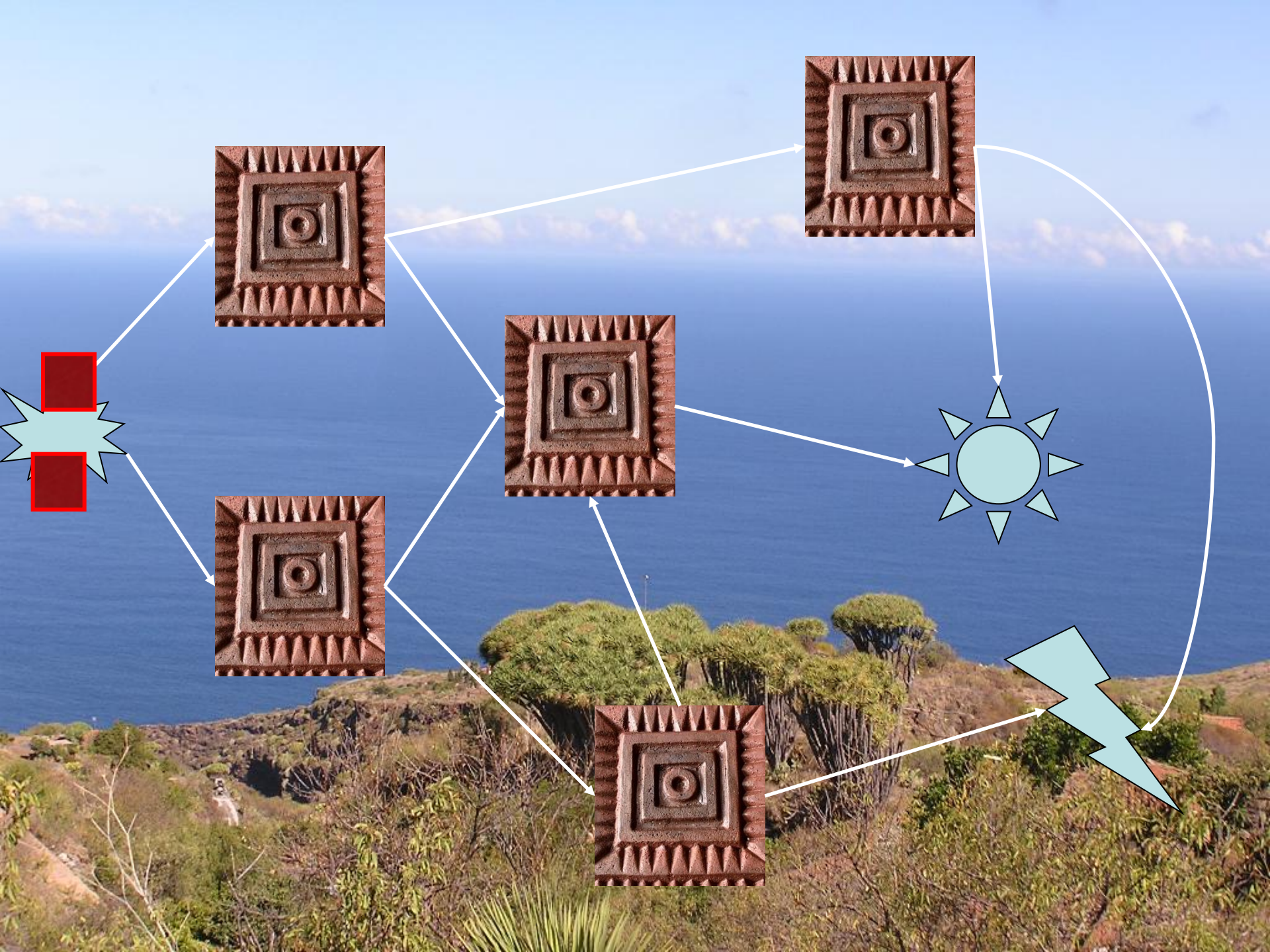


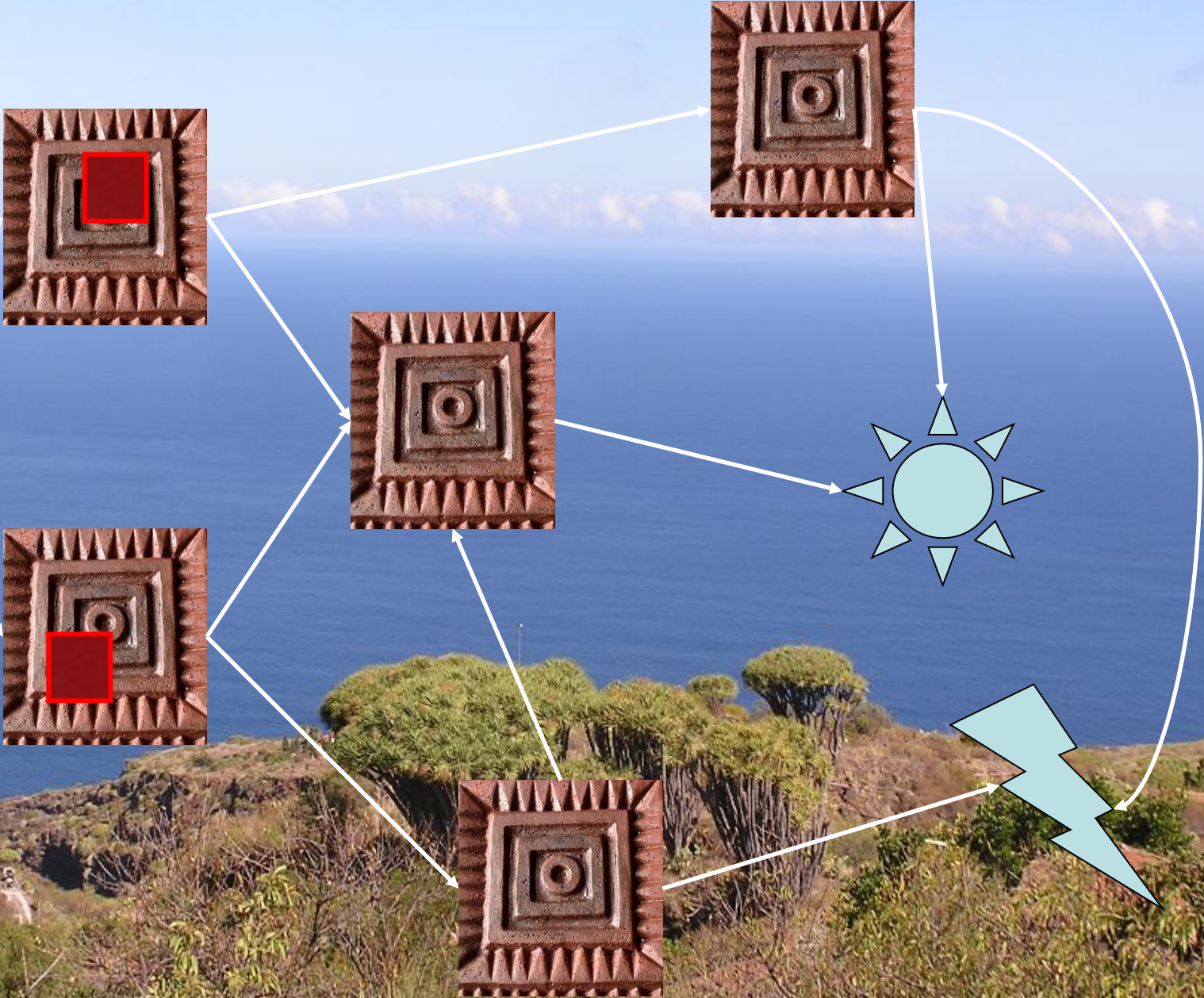
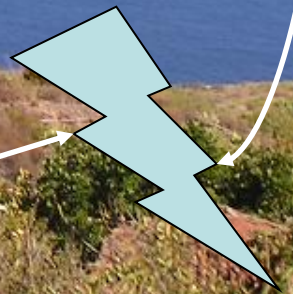
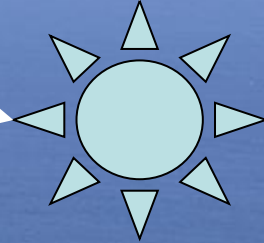
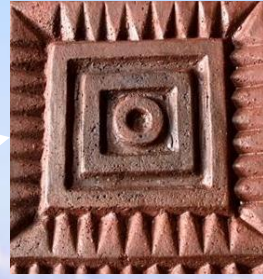
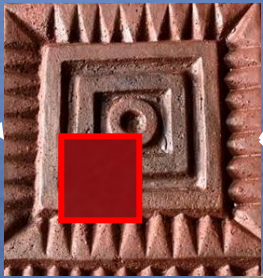
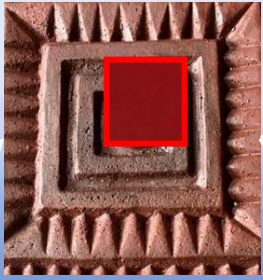
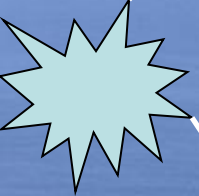
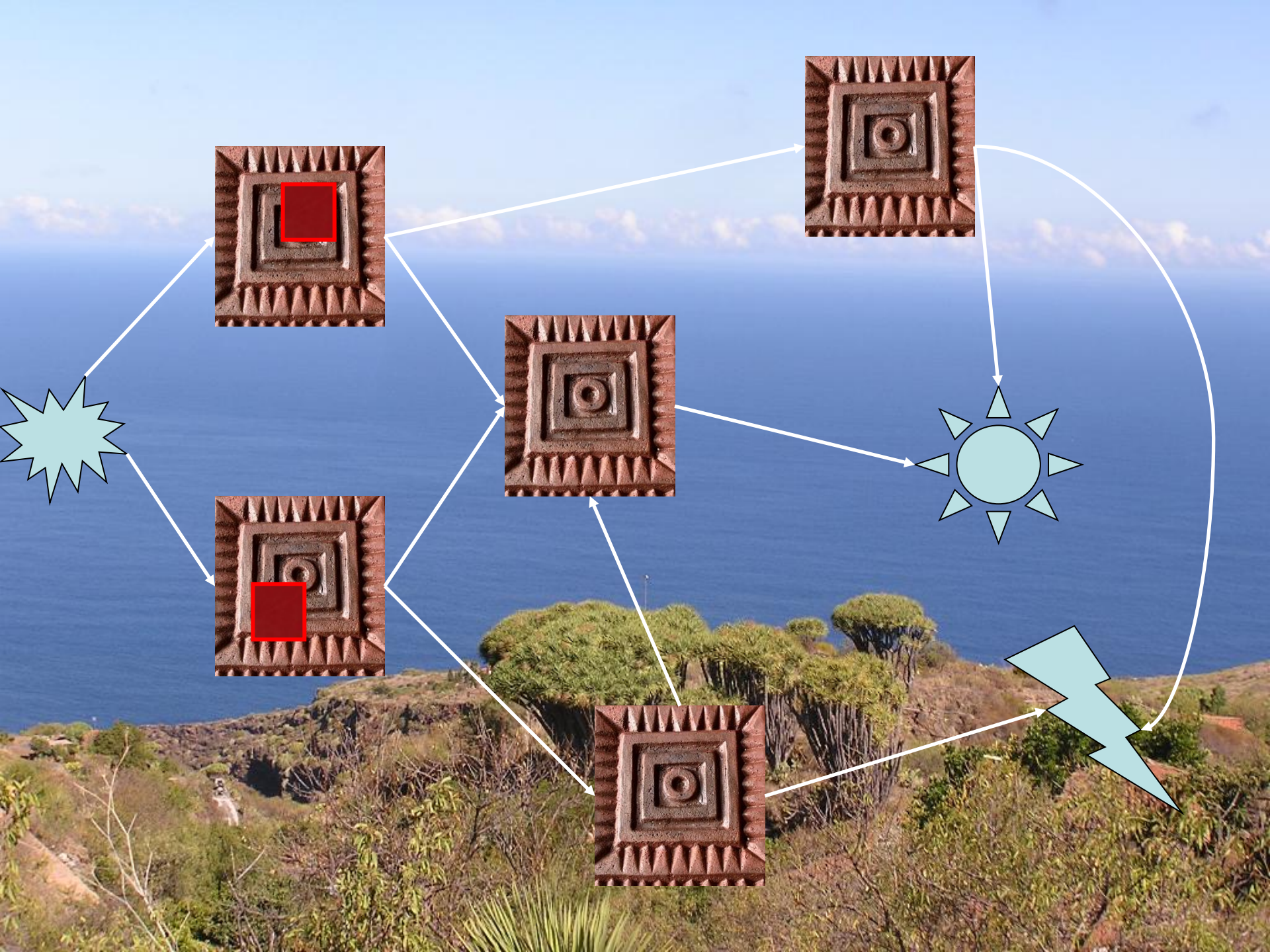


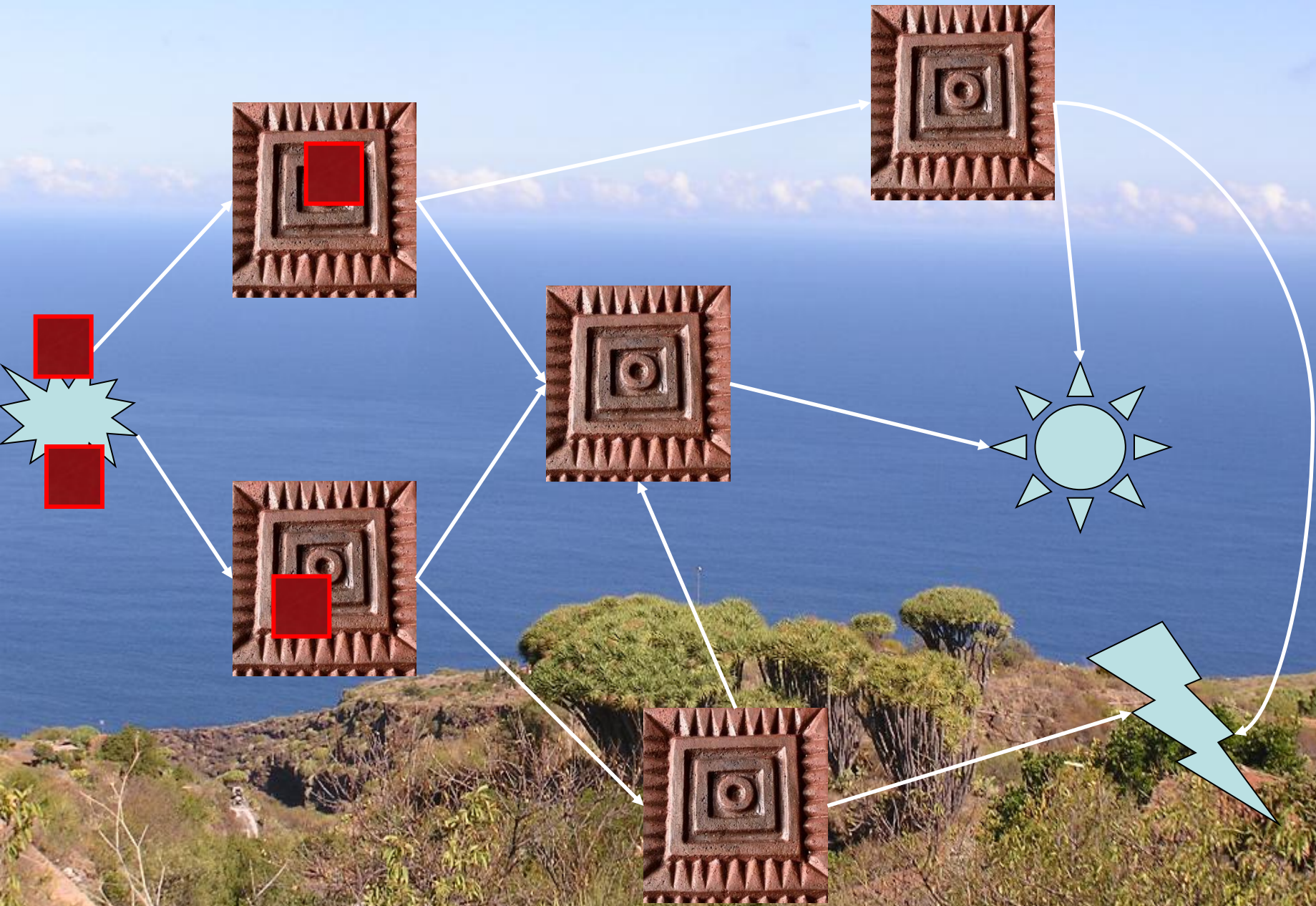
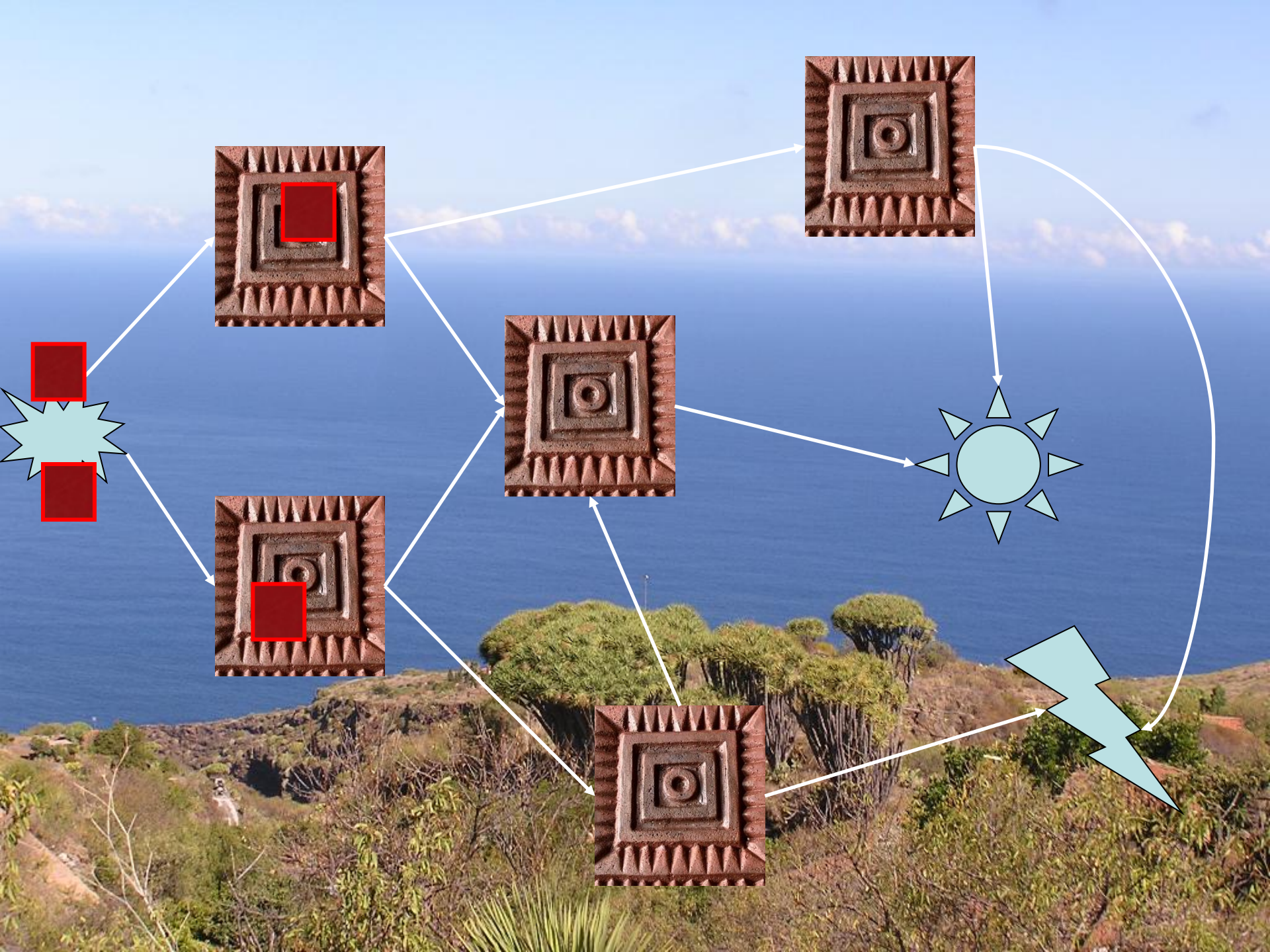
# Abaco probabilístico(Engel, 1975)

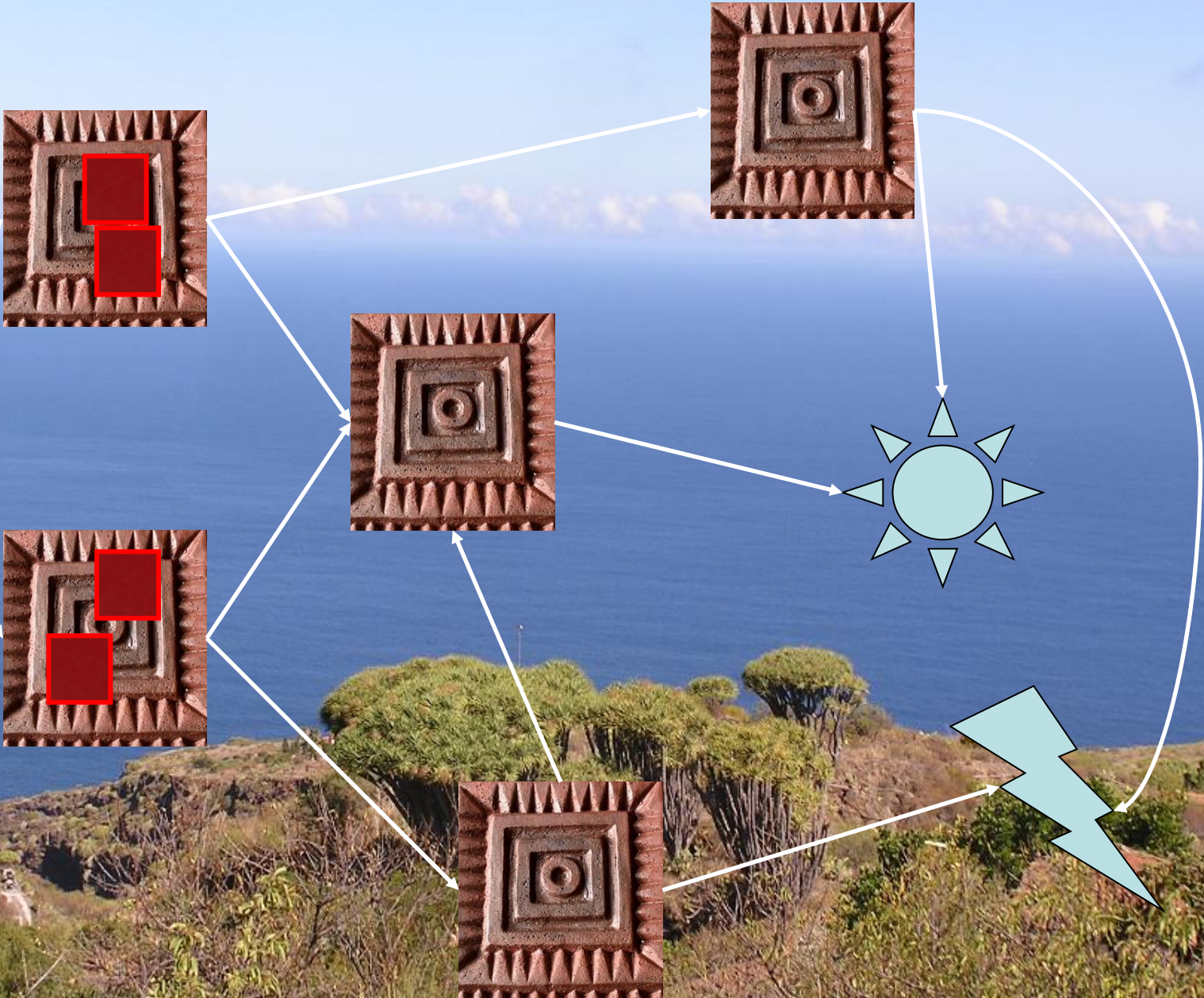
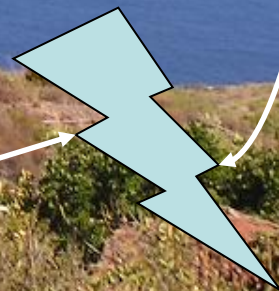
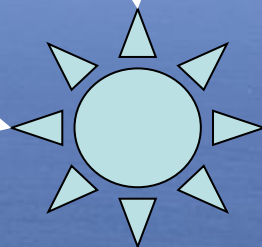
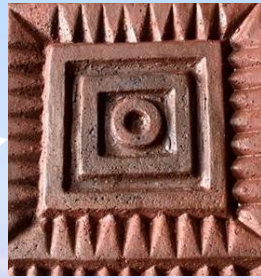
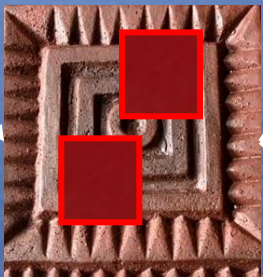
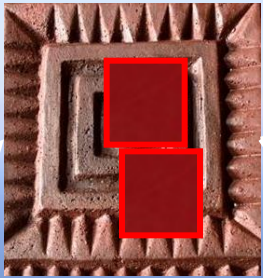
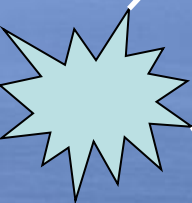
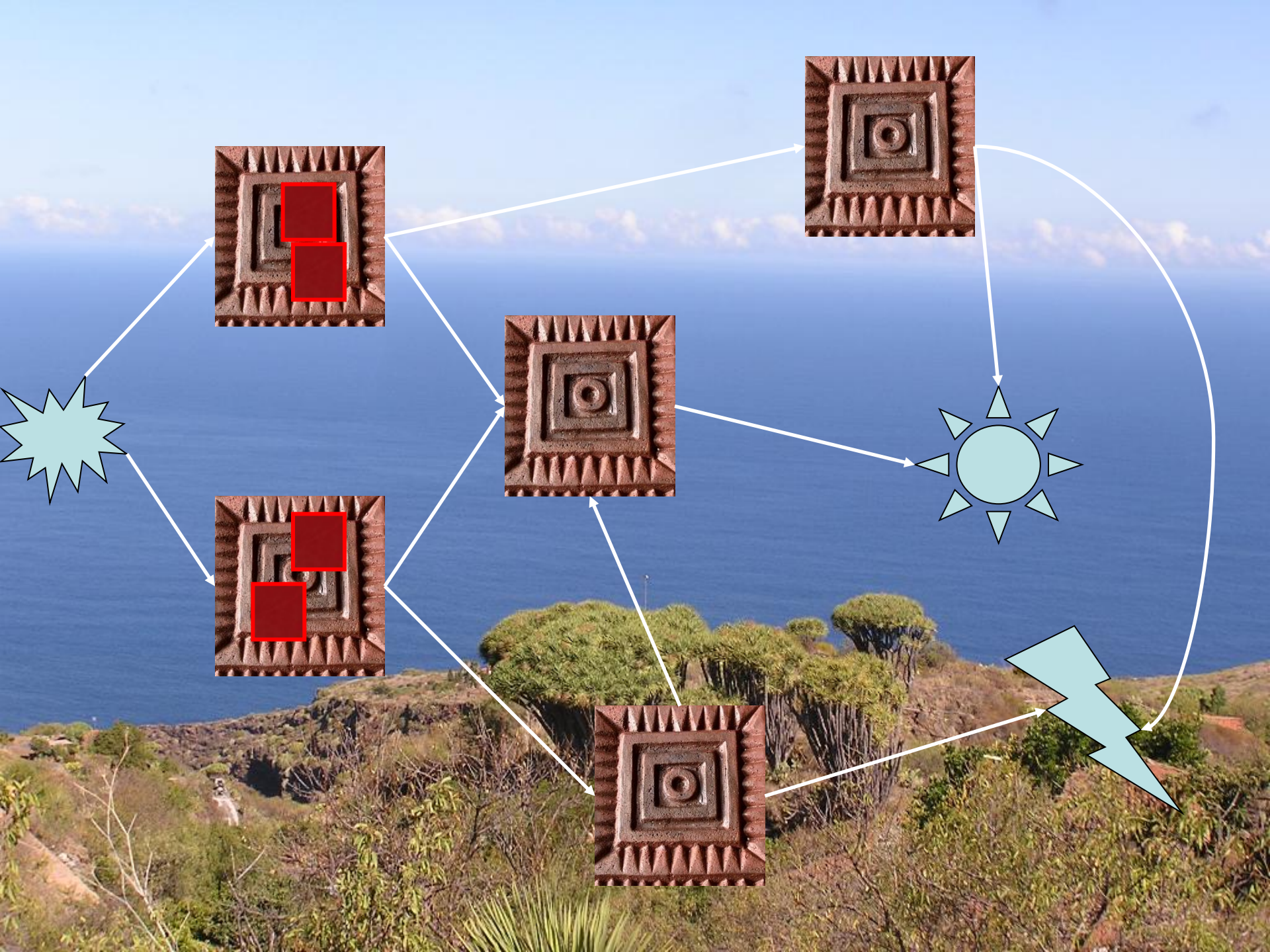
*En cada bifurcación se han de repartir las fichas de forma proporcional a la probabilidad de que se tome cada bifurcación. No habrá movimiento hasta que no se disponga del número de fichas necesario. La idea es ir consiguiendo situaciones de estabilidad, es decir, en cada bifurcación se ha de tener un número de fichas igual al número de caminos que de ella parten para poder mover las fichas.*

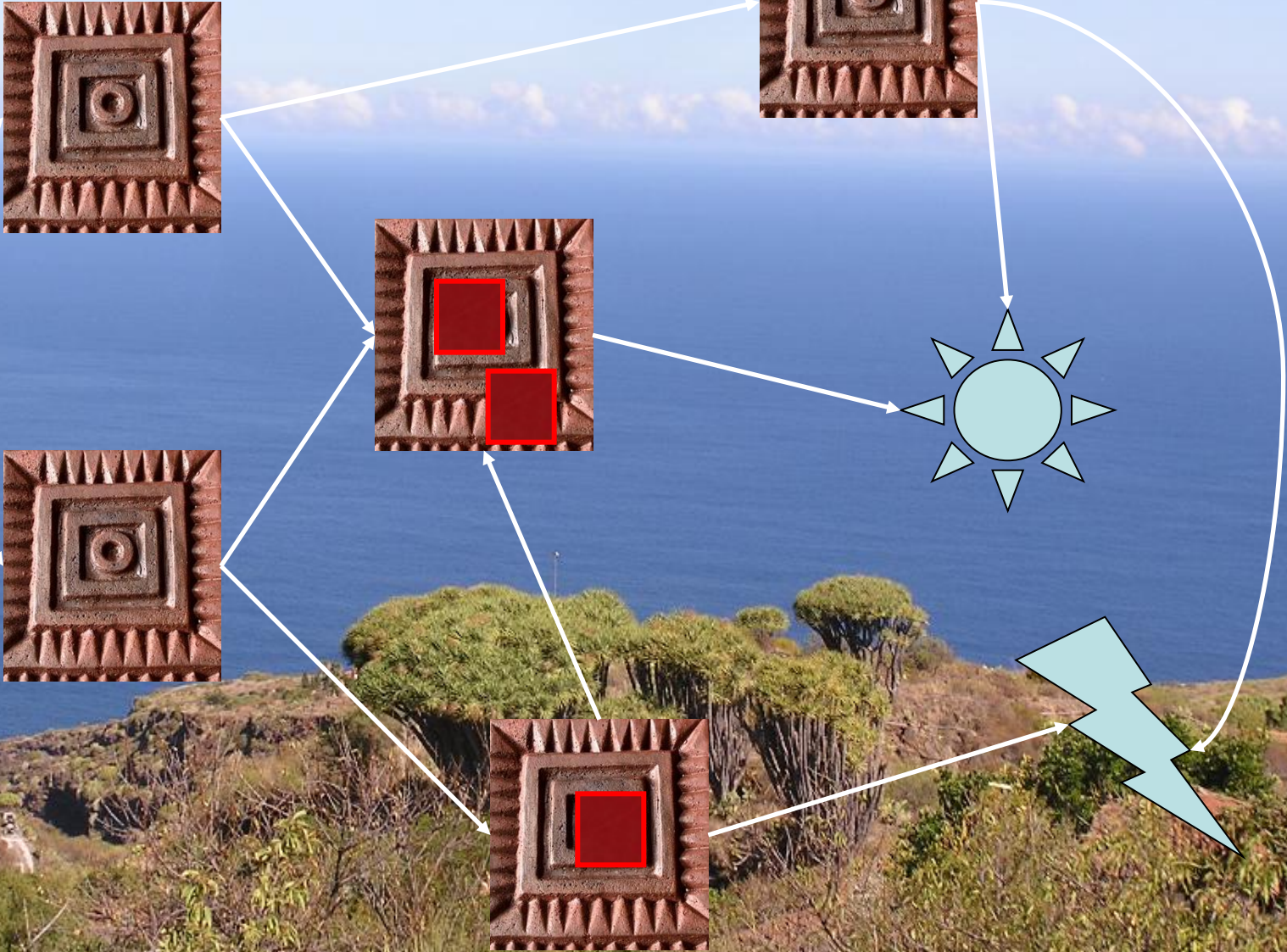
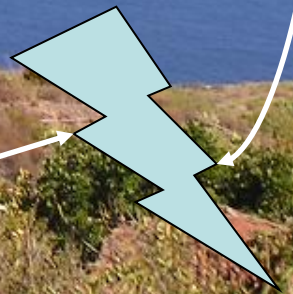
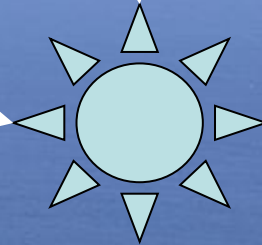
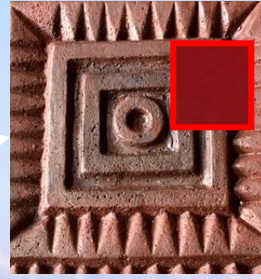
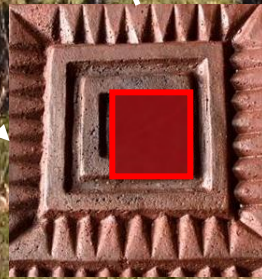
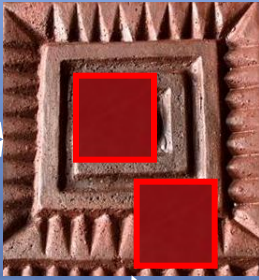
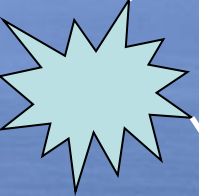
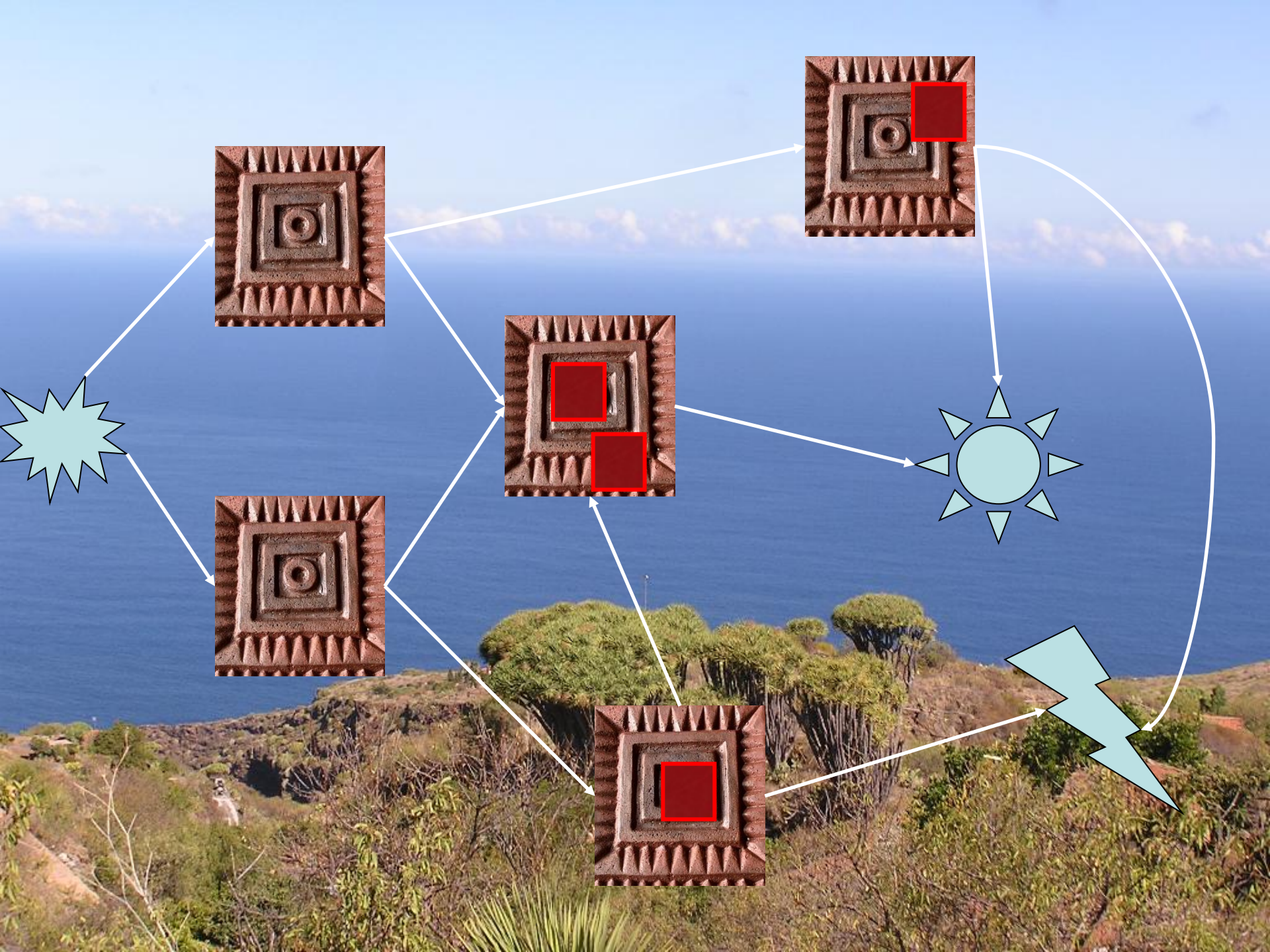
**TABLERON**

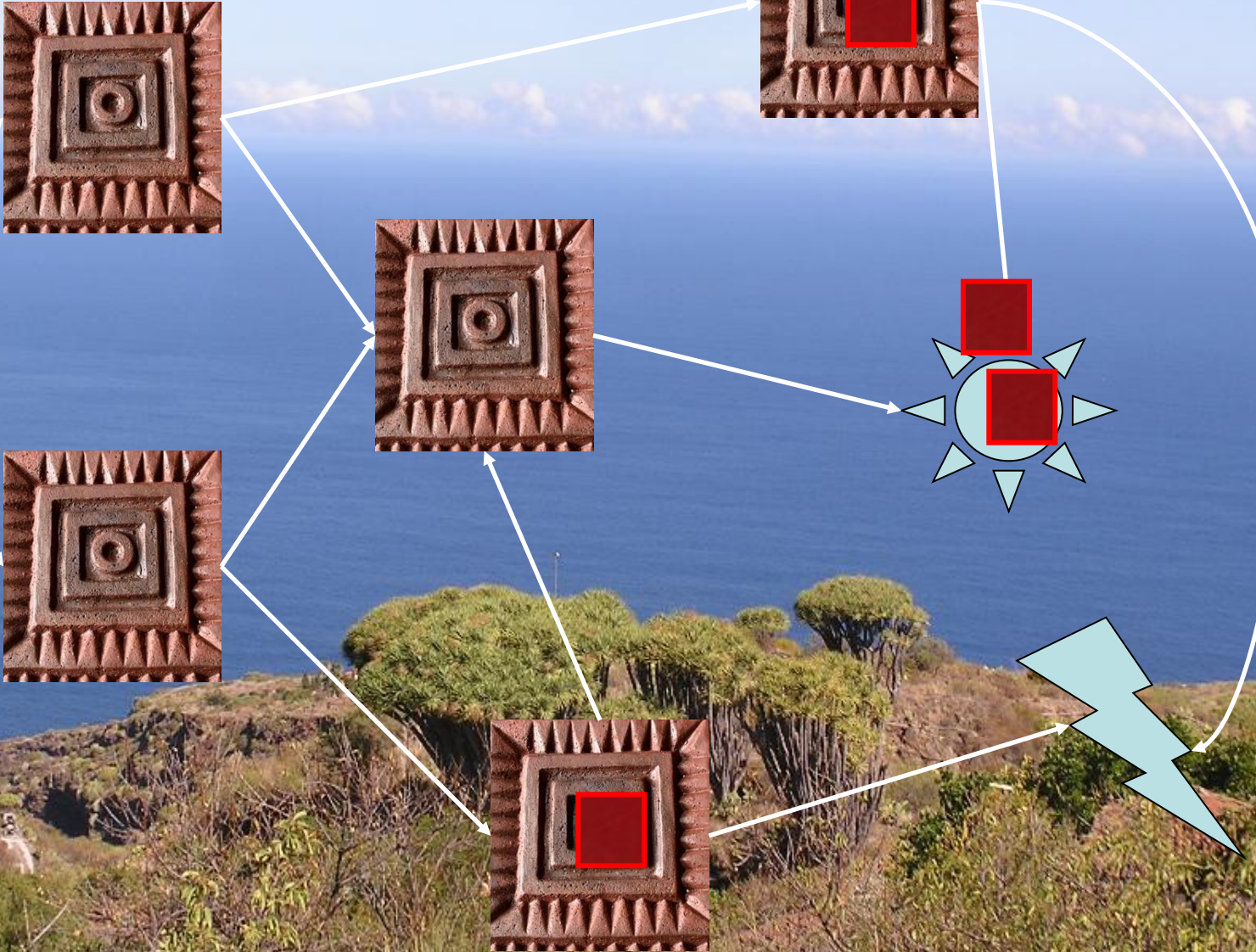
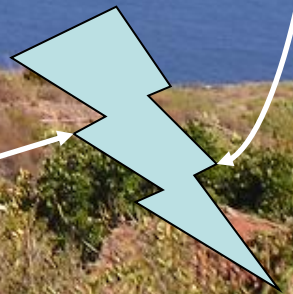
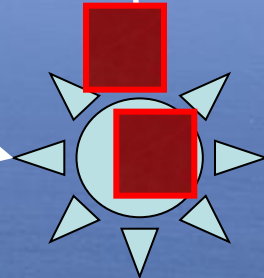
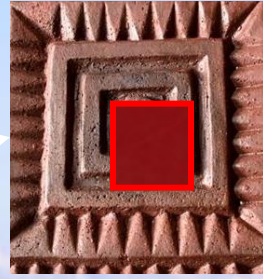
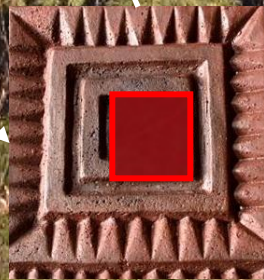
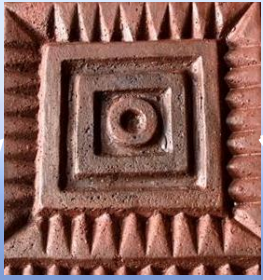
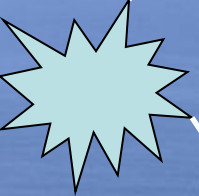
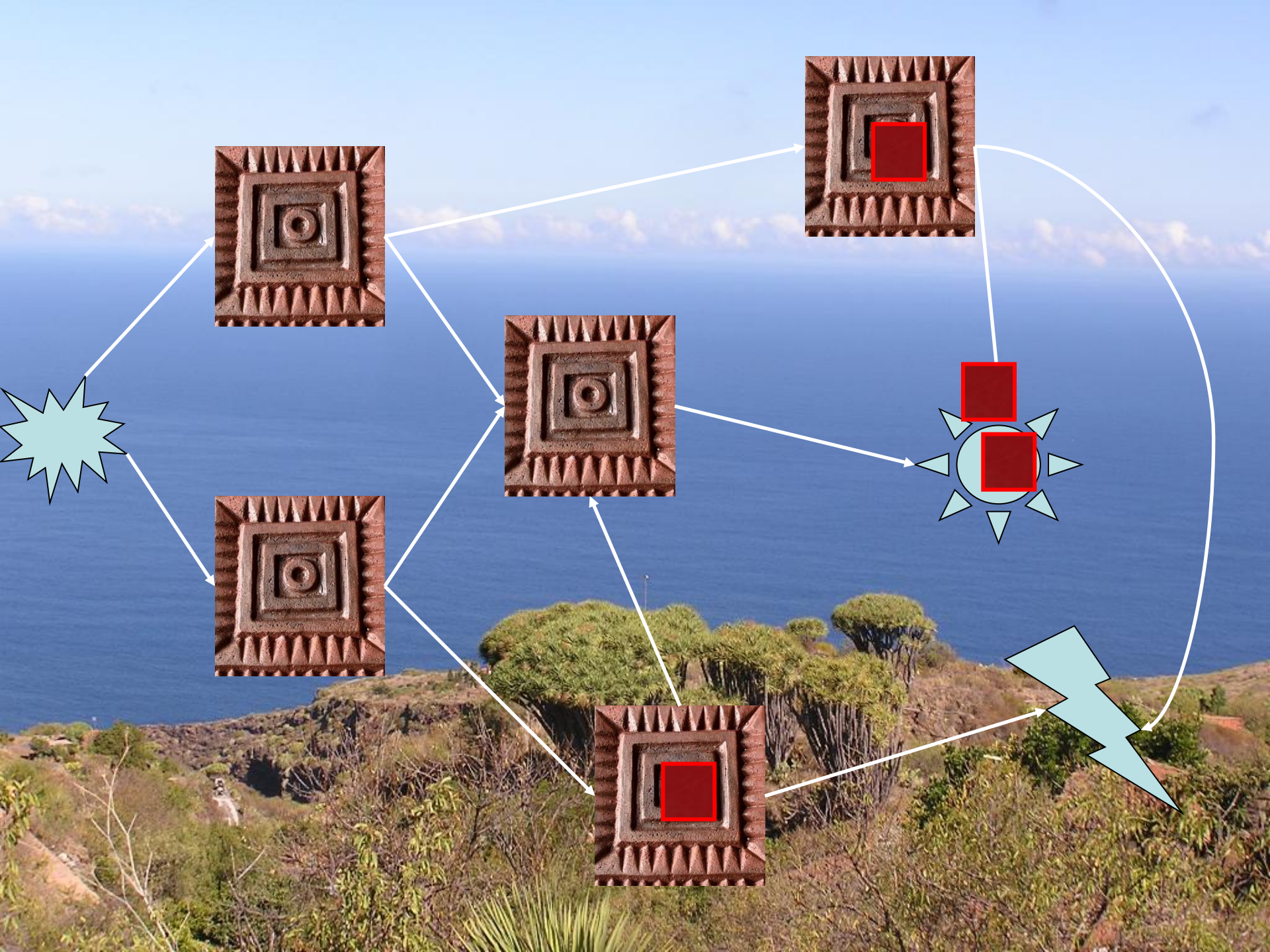


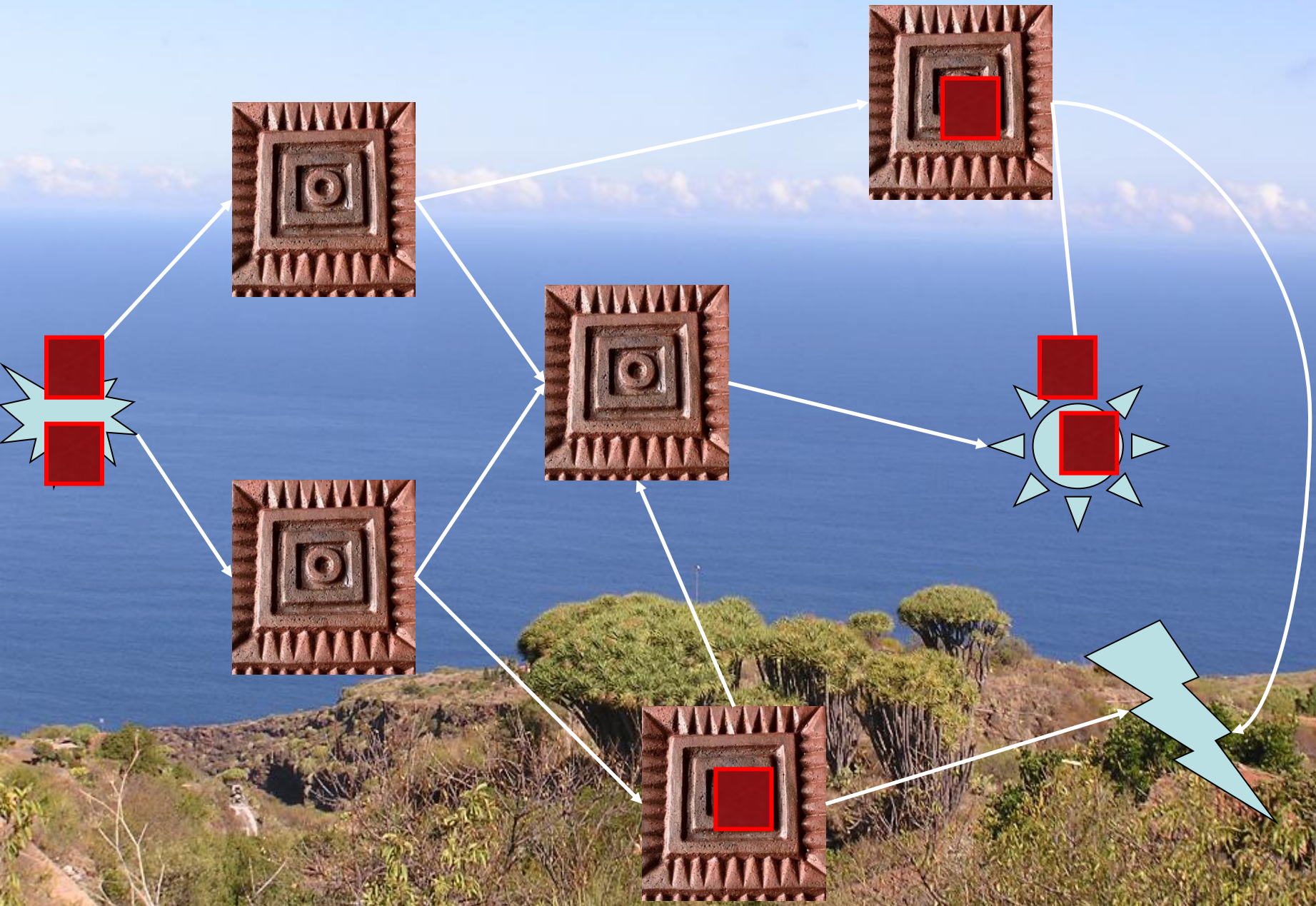
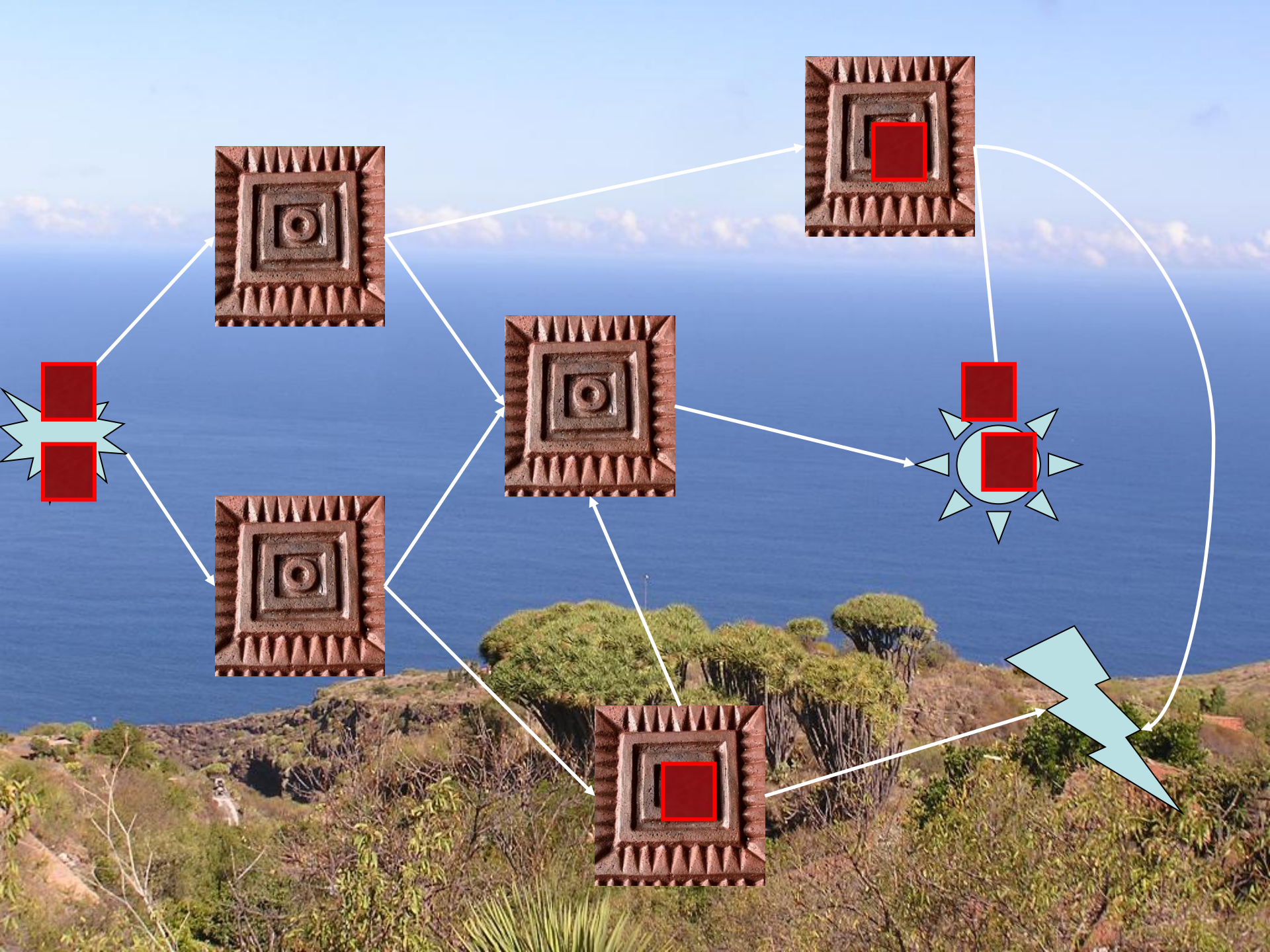


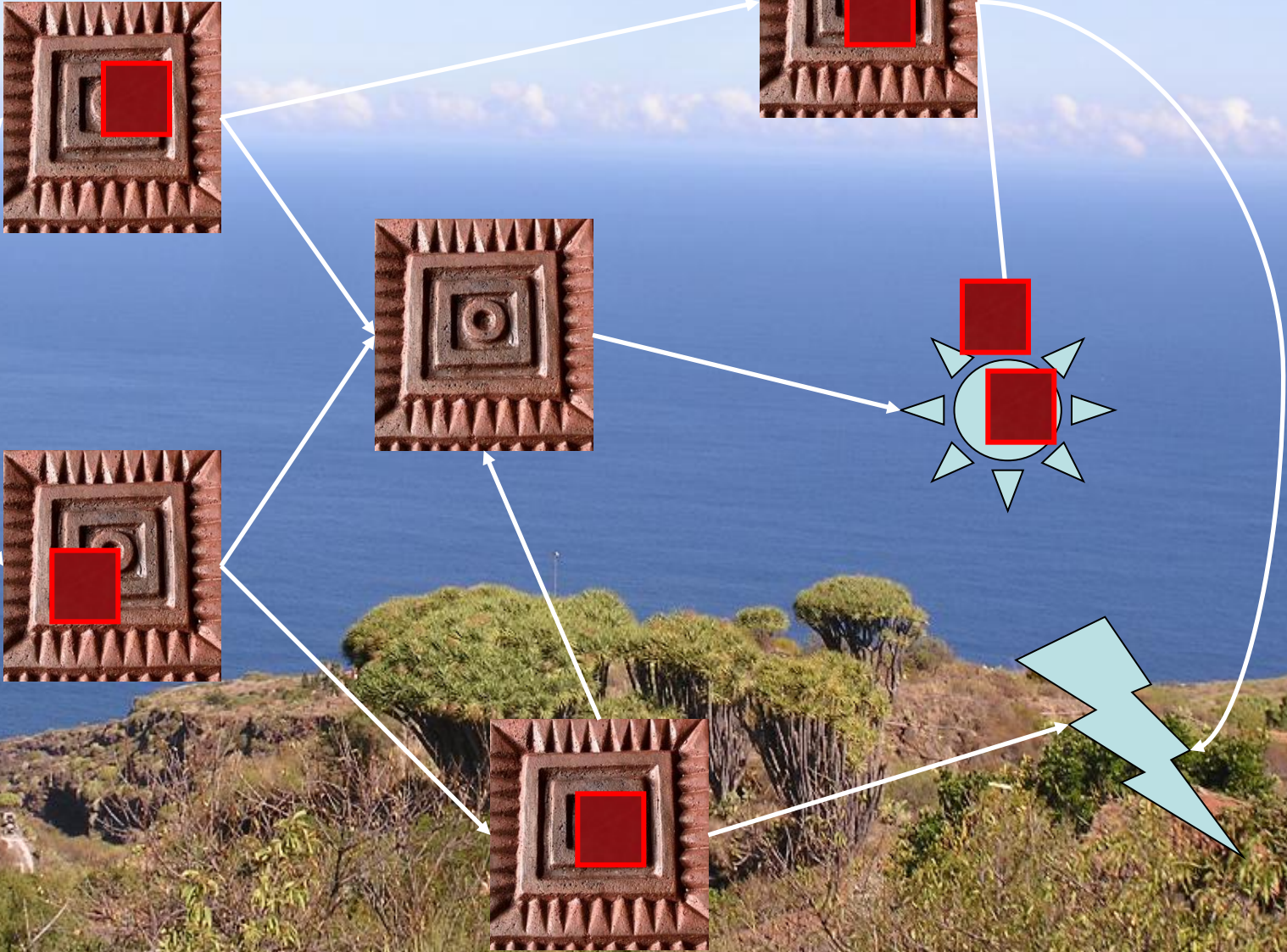
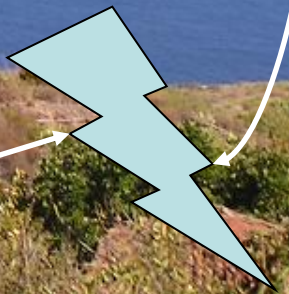
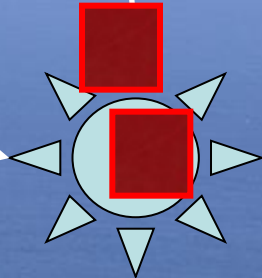
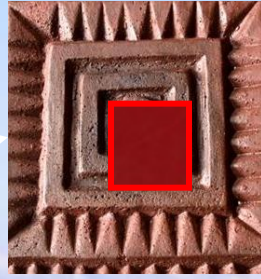
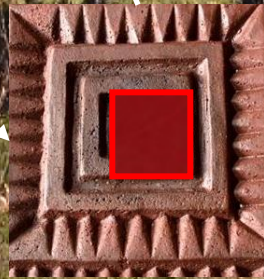
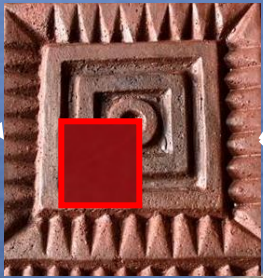
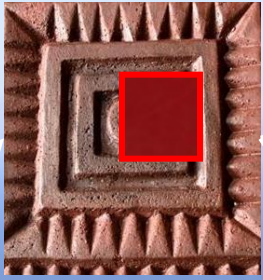
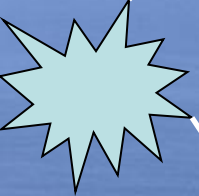
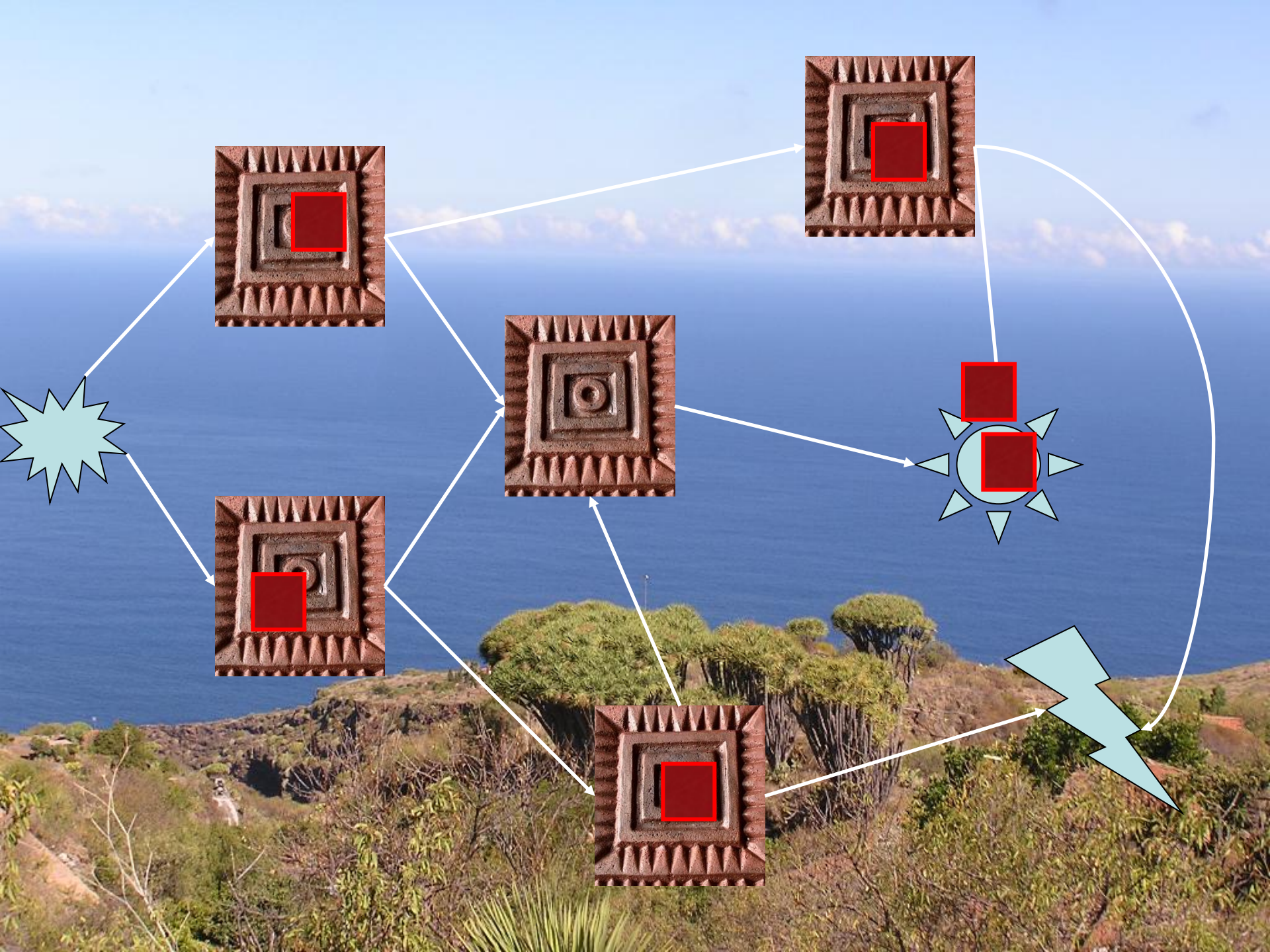


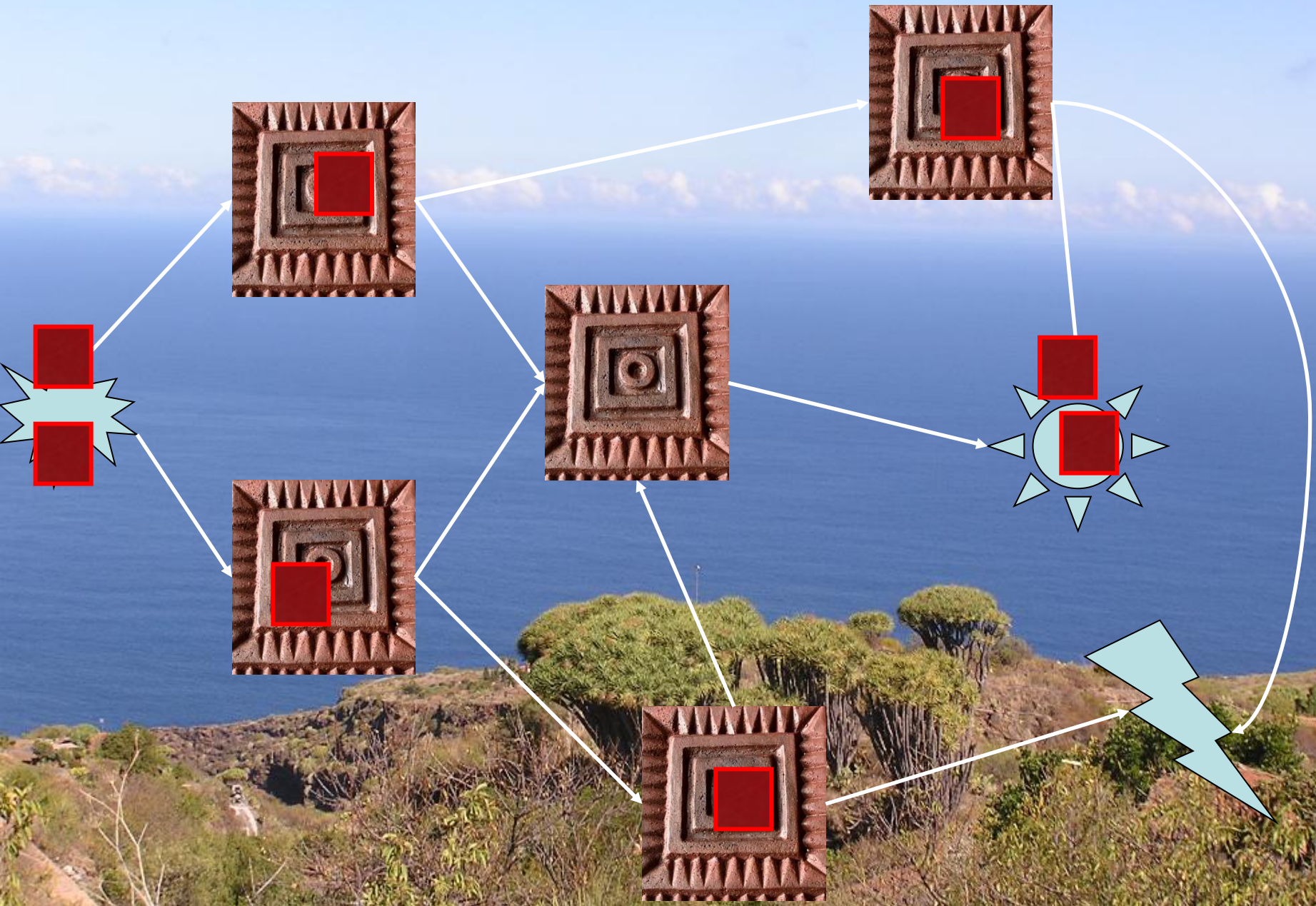
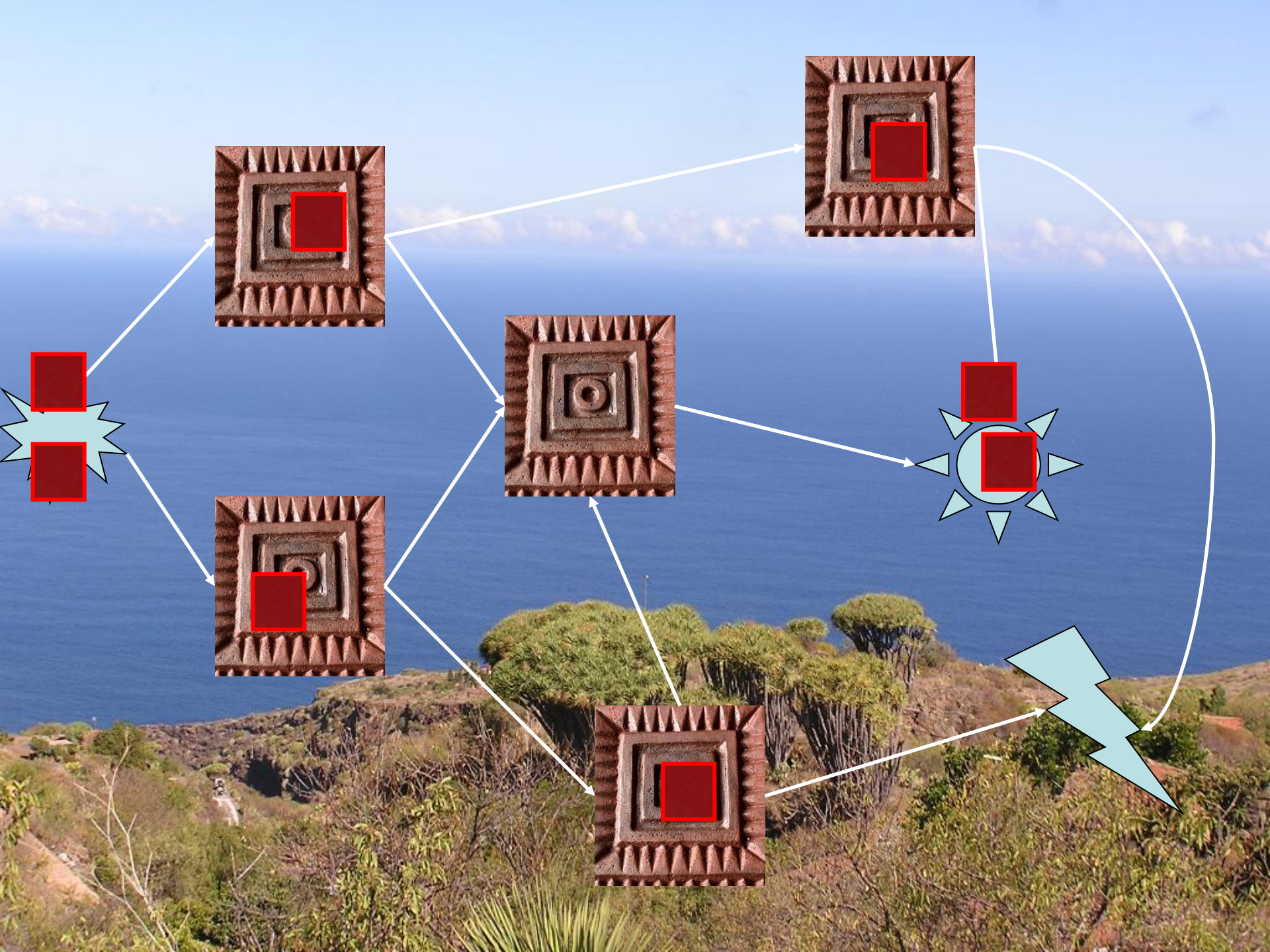


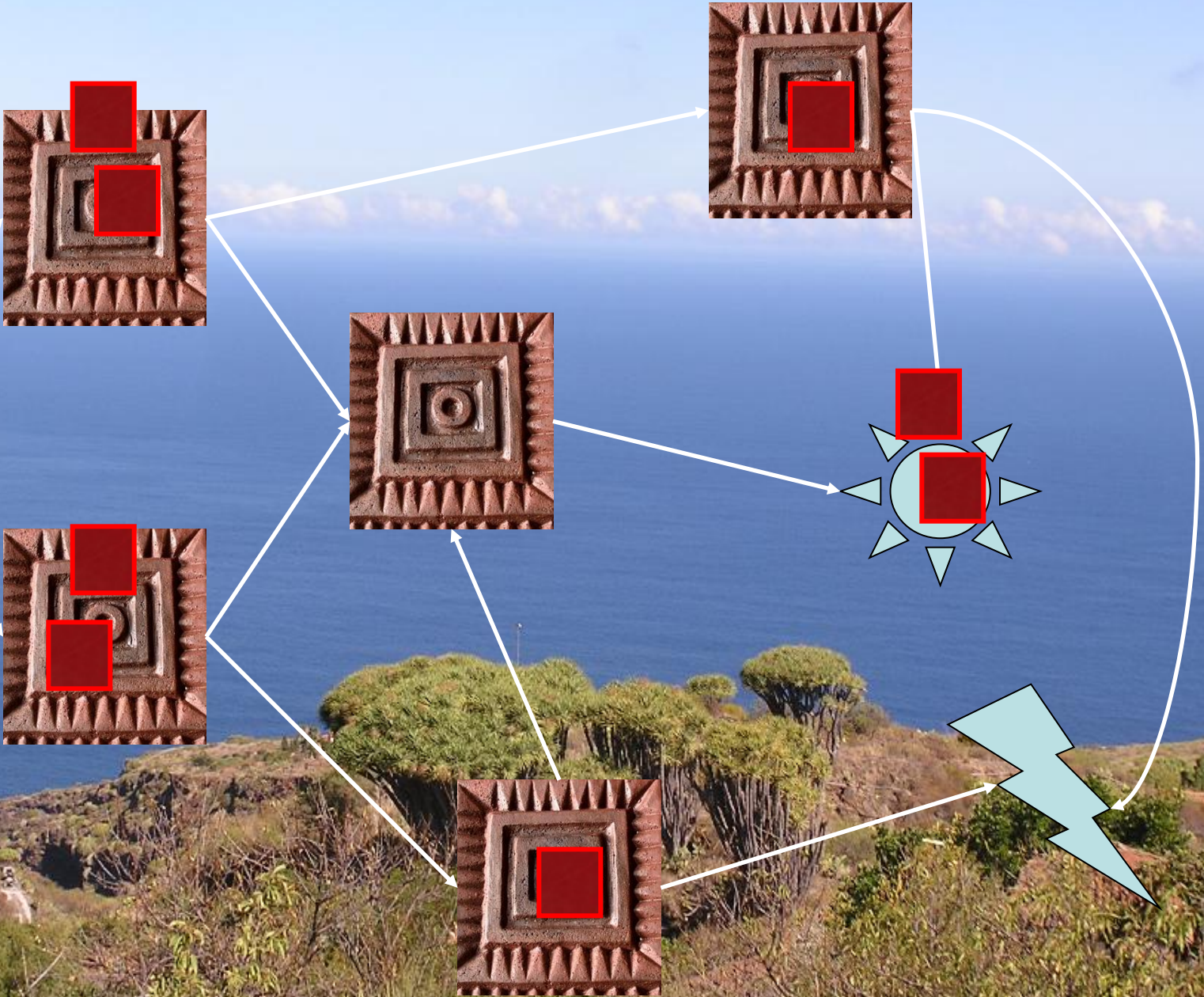
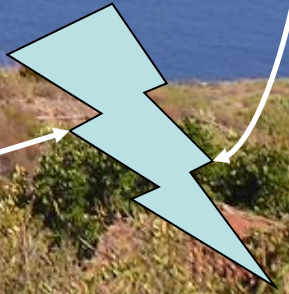
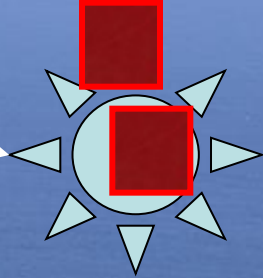
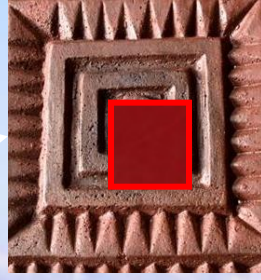
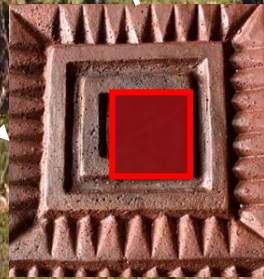
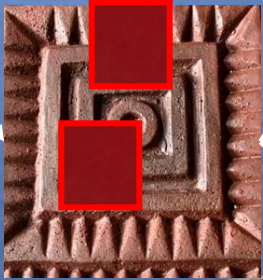
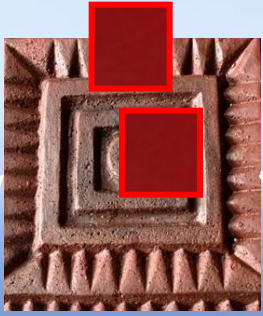
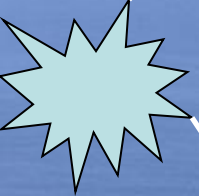
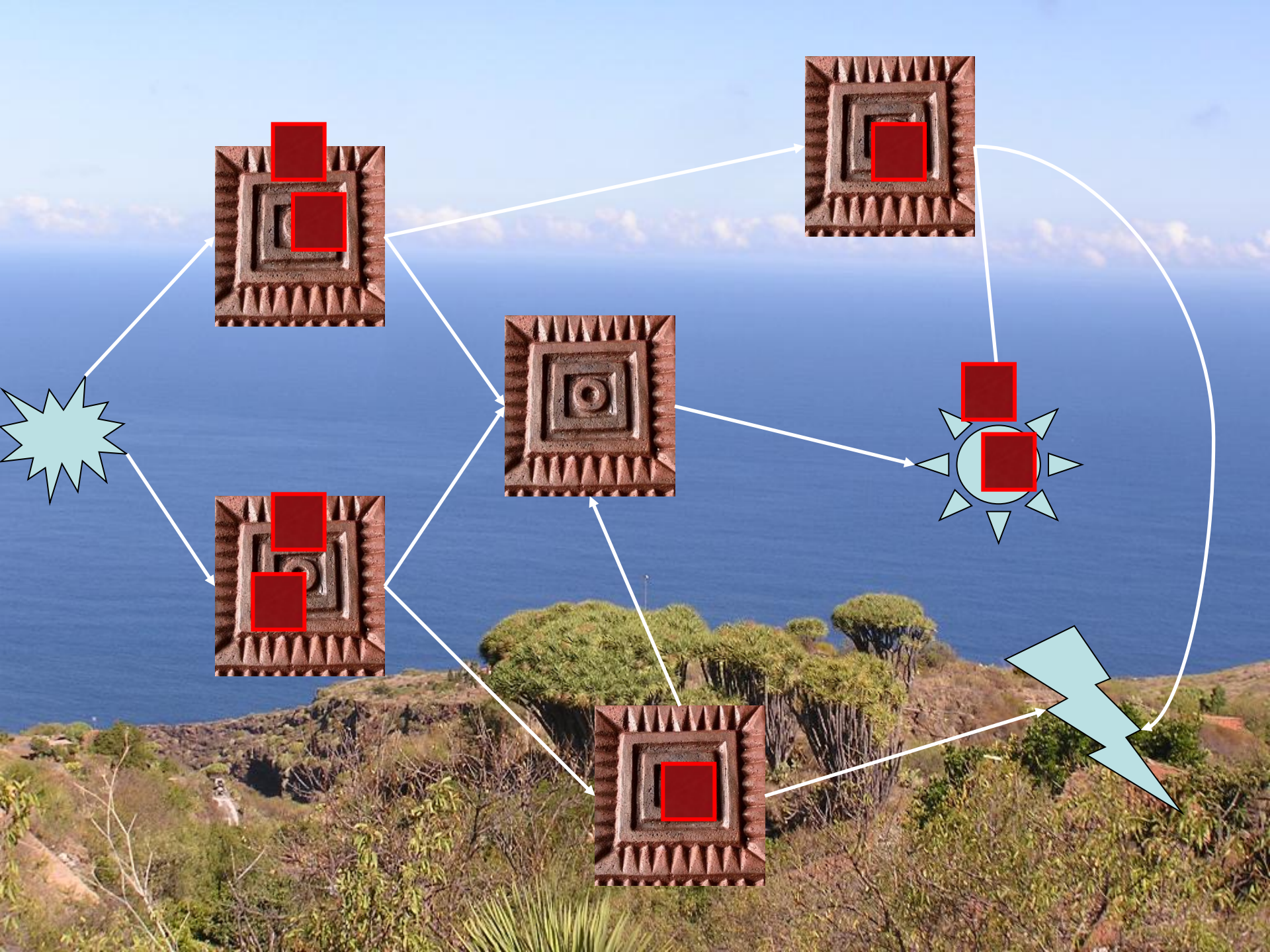


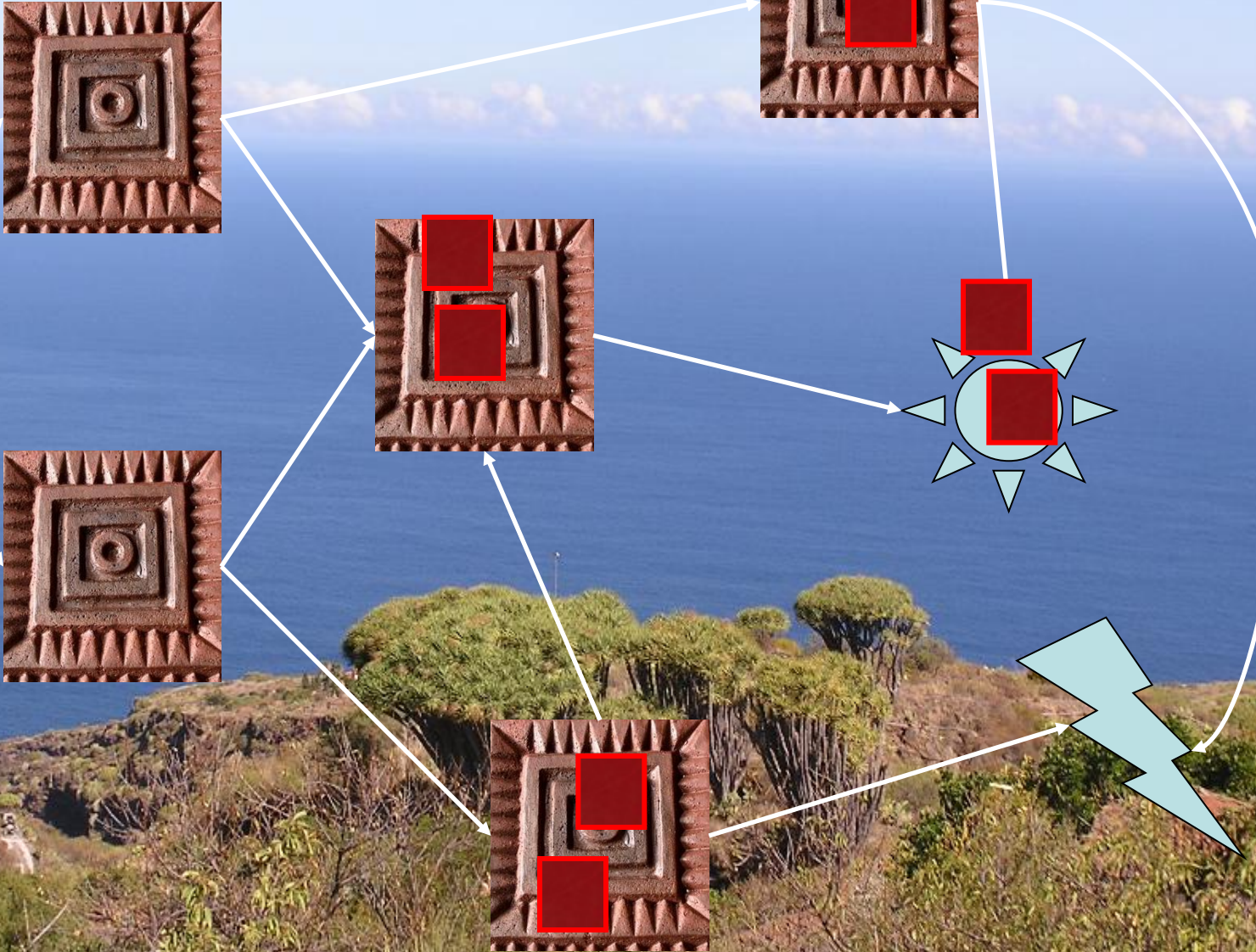
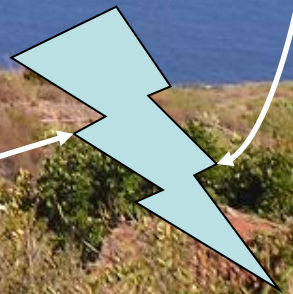
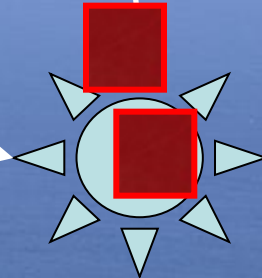
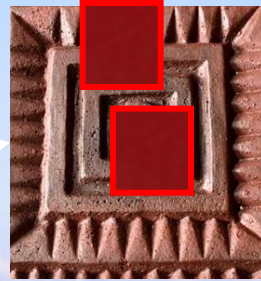
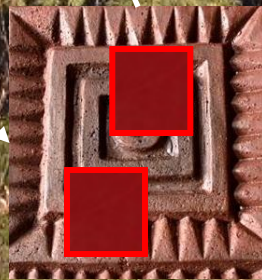
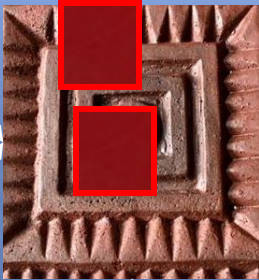
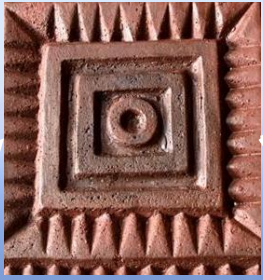
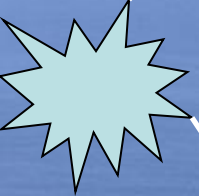
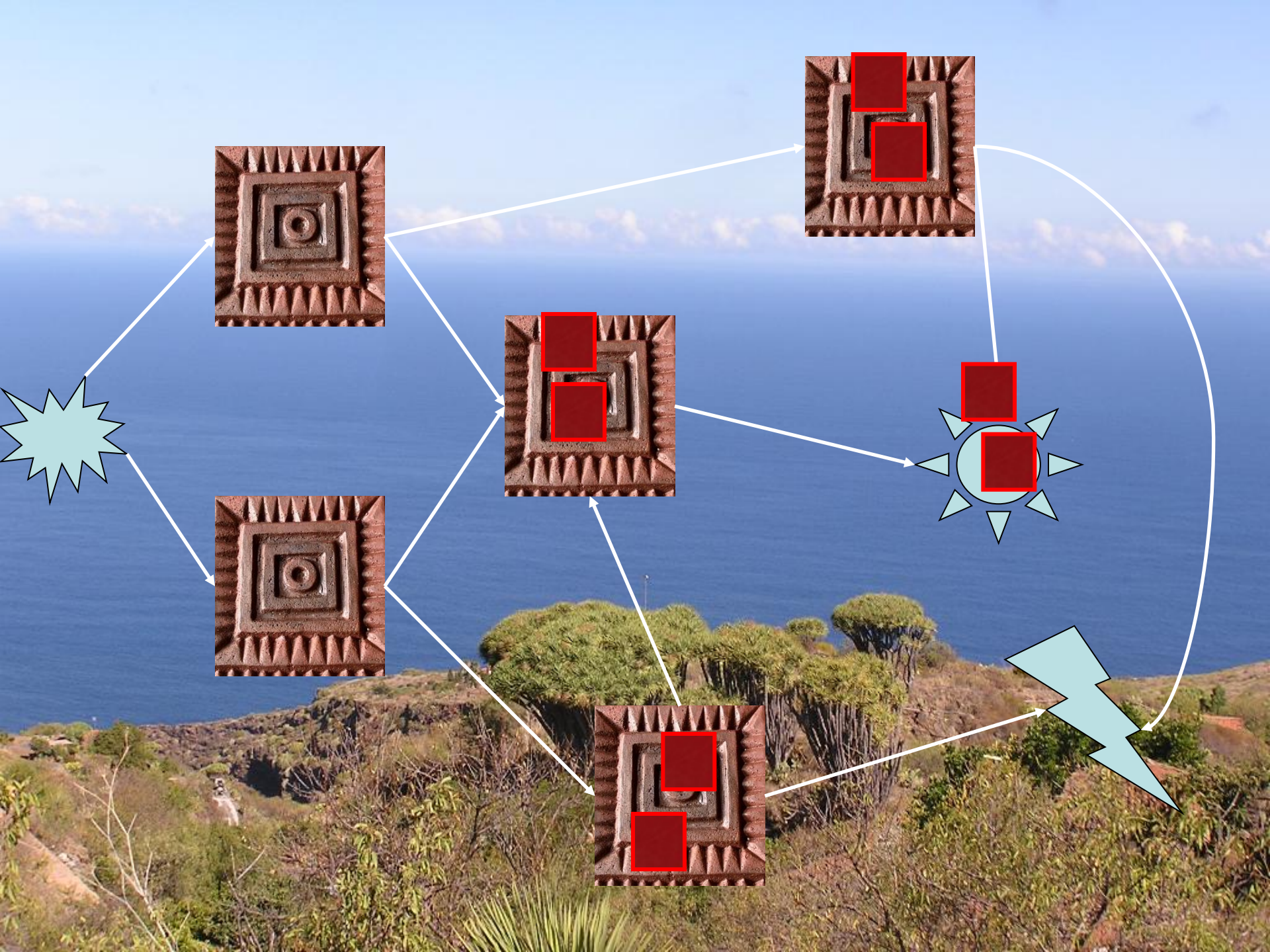




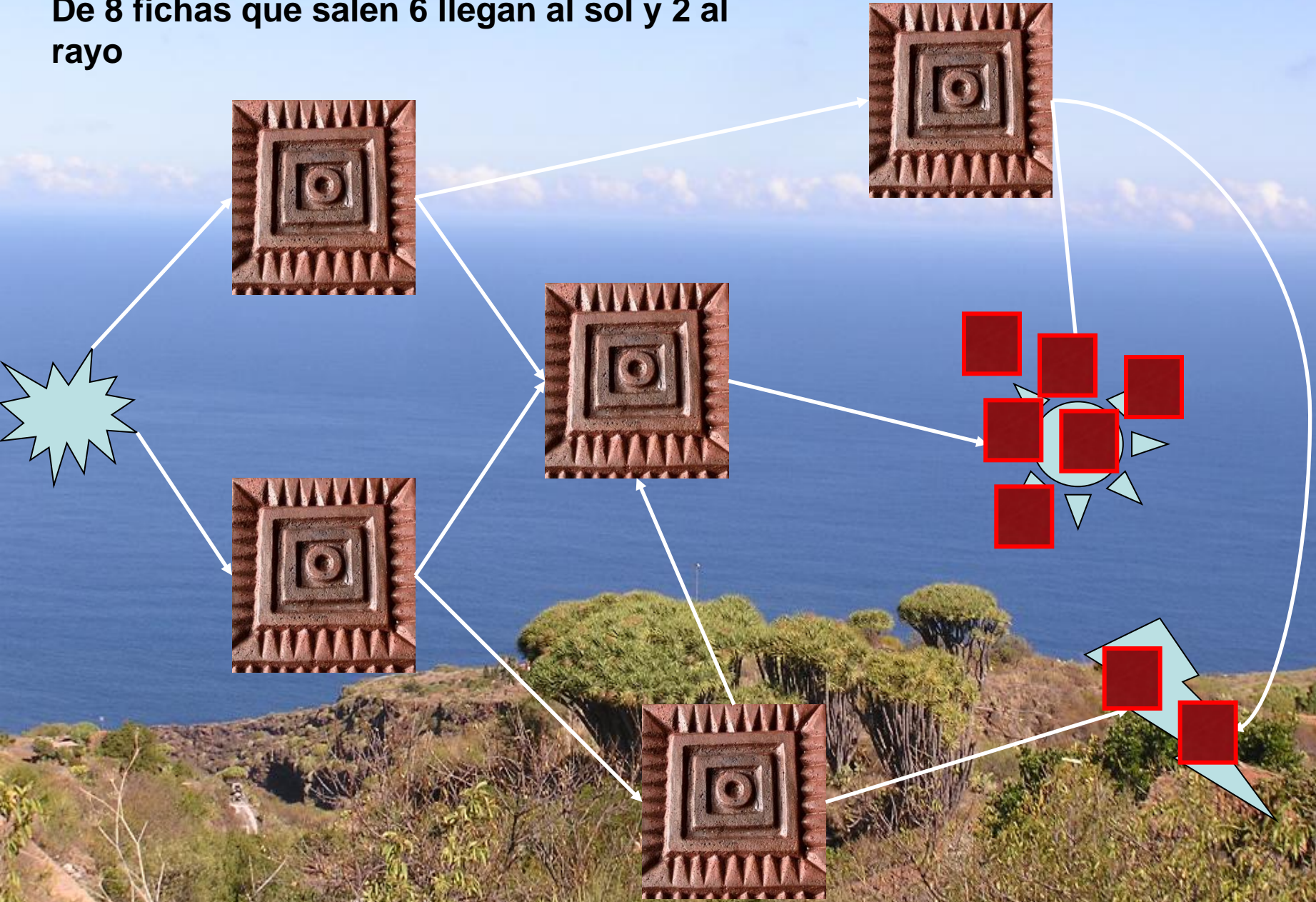




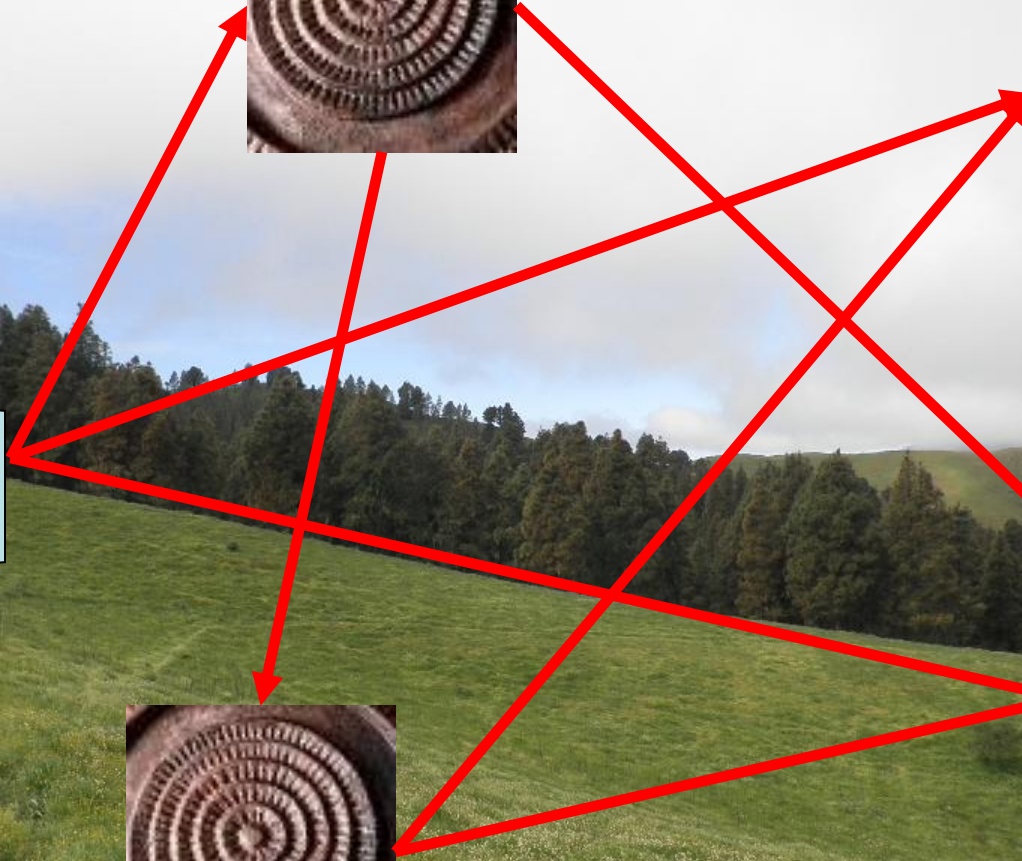
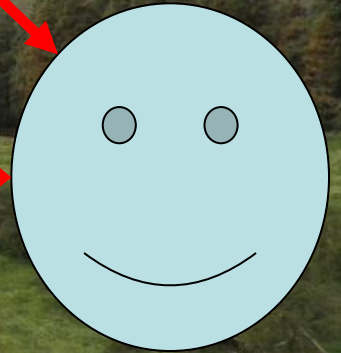
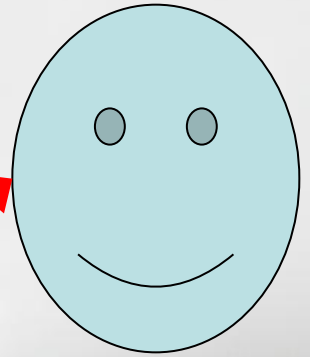
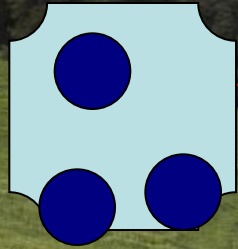
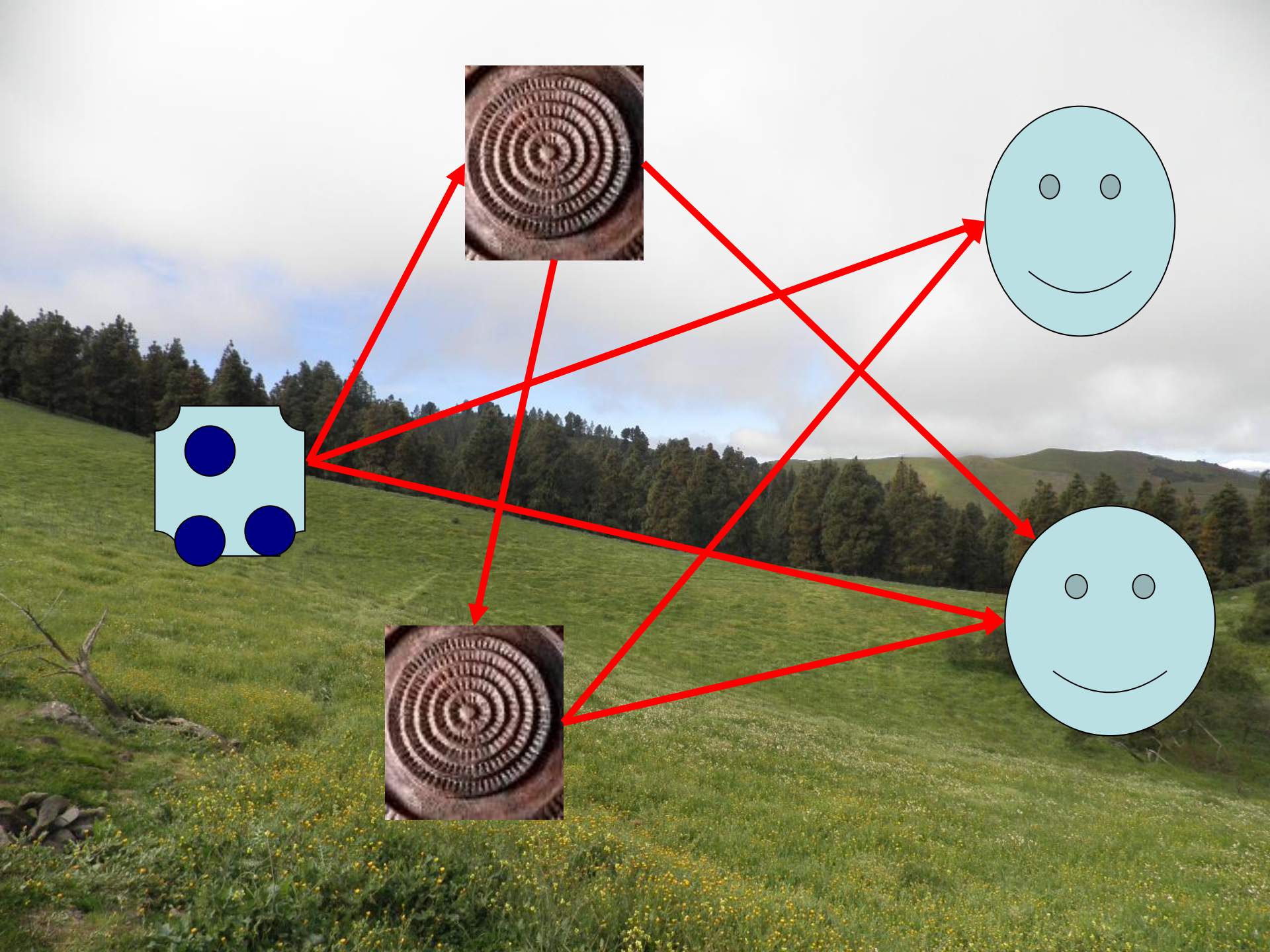


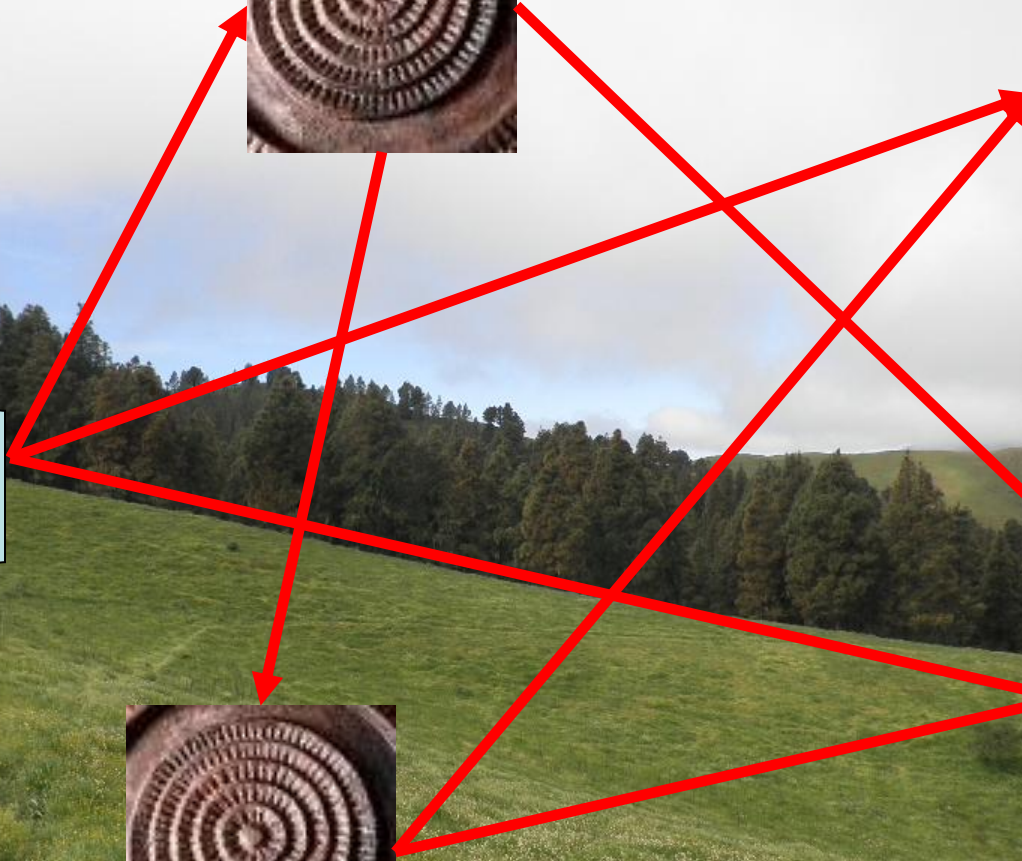
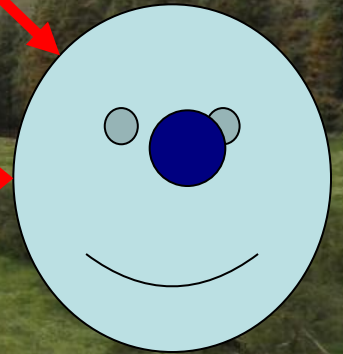
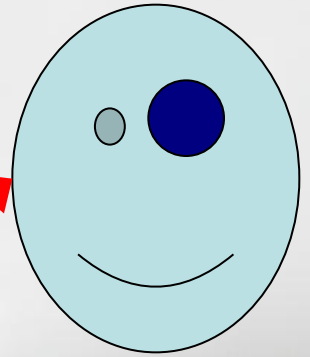
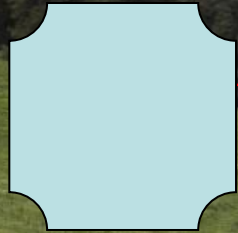
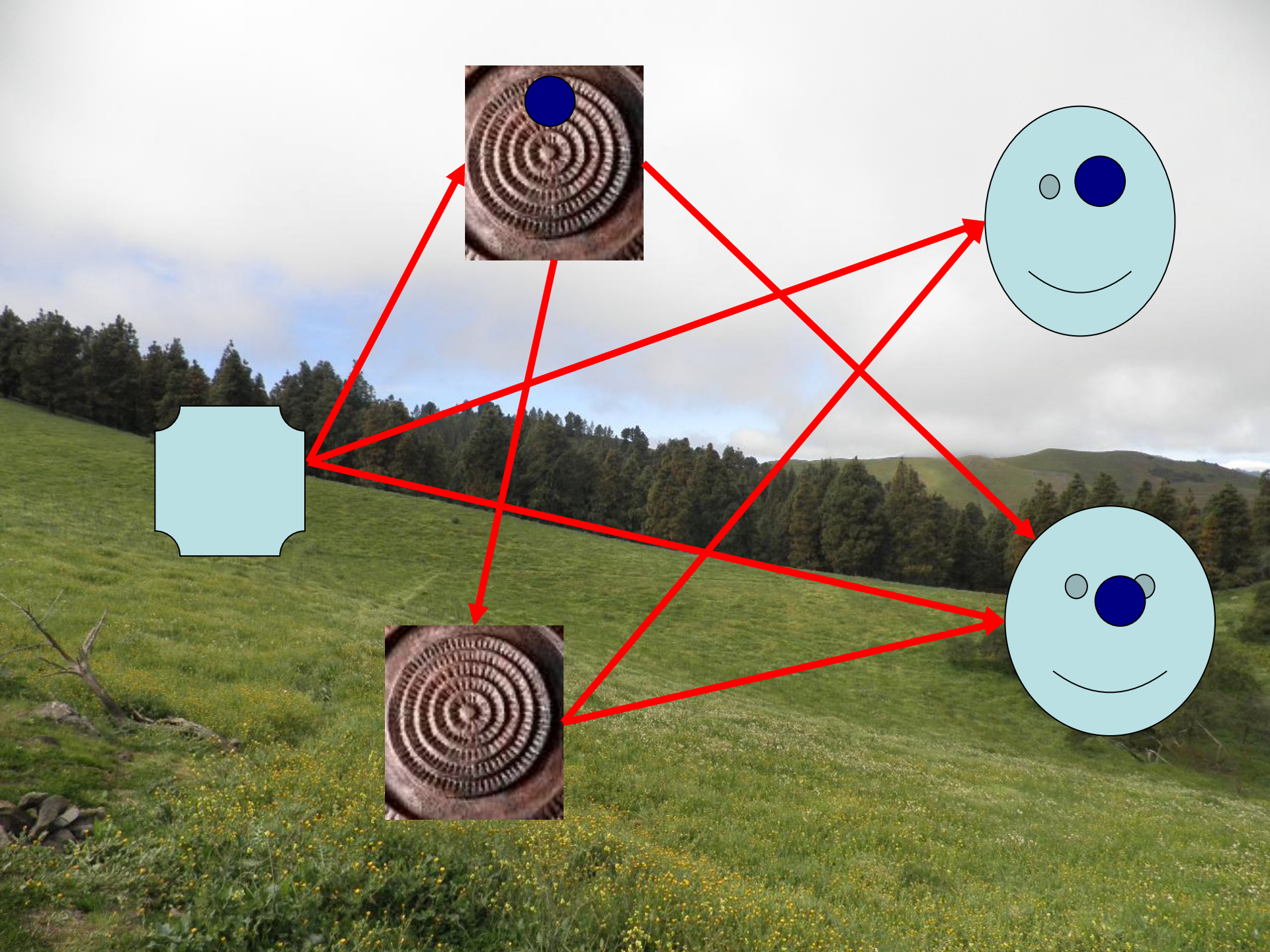


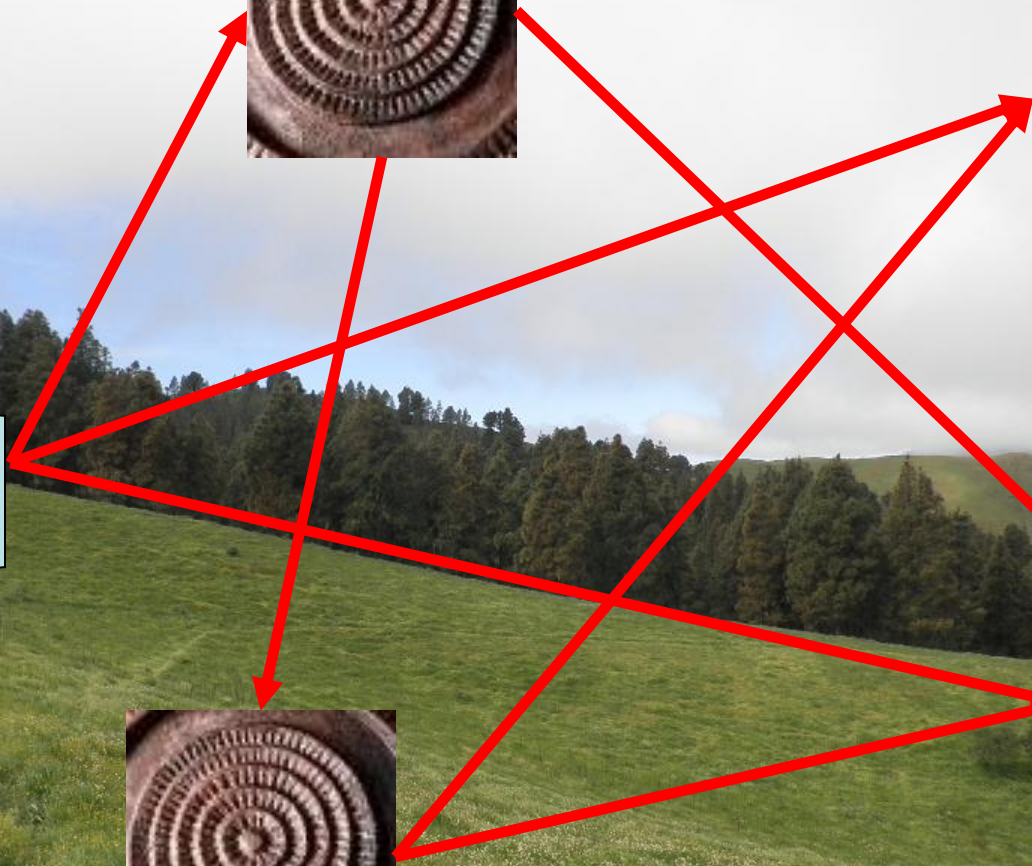
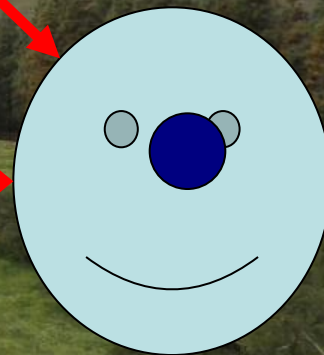
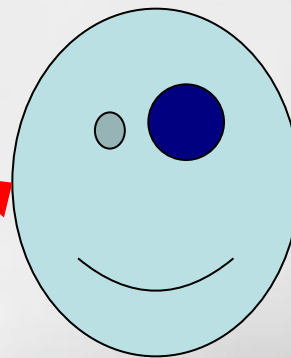
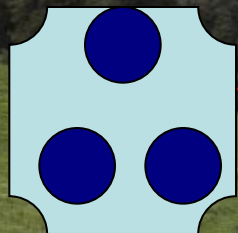
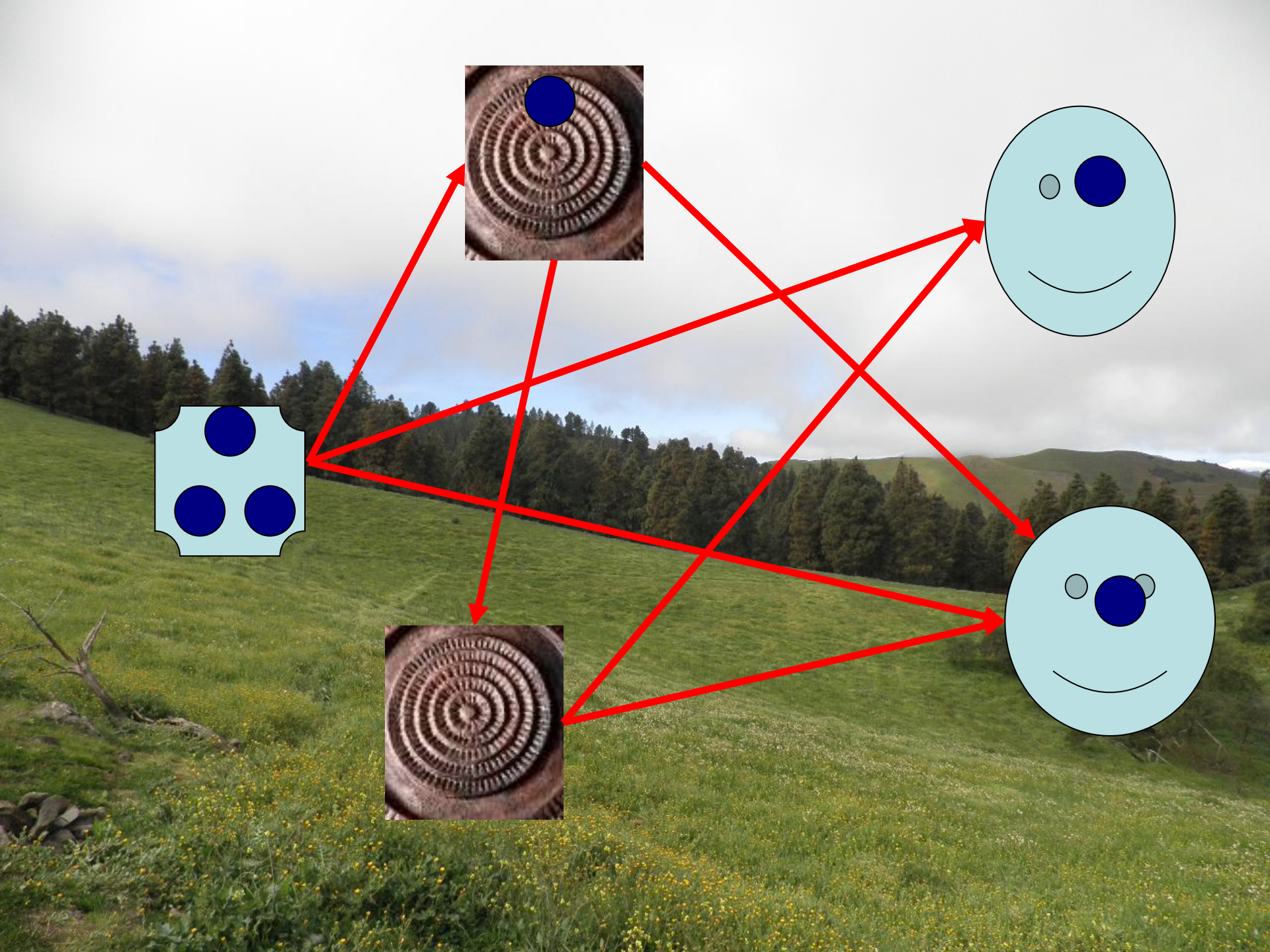
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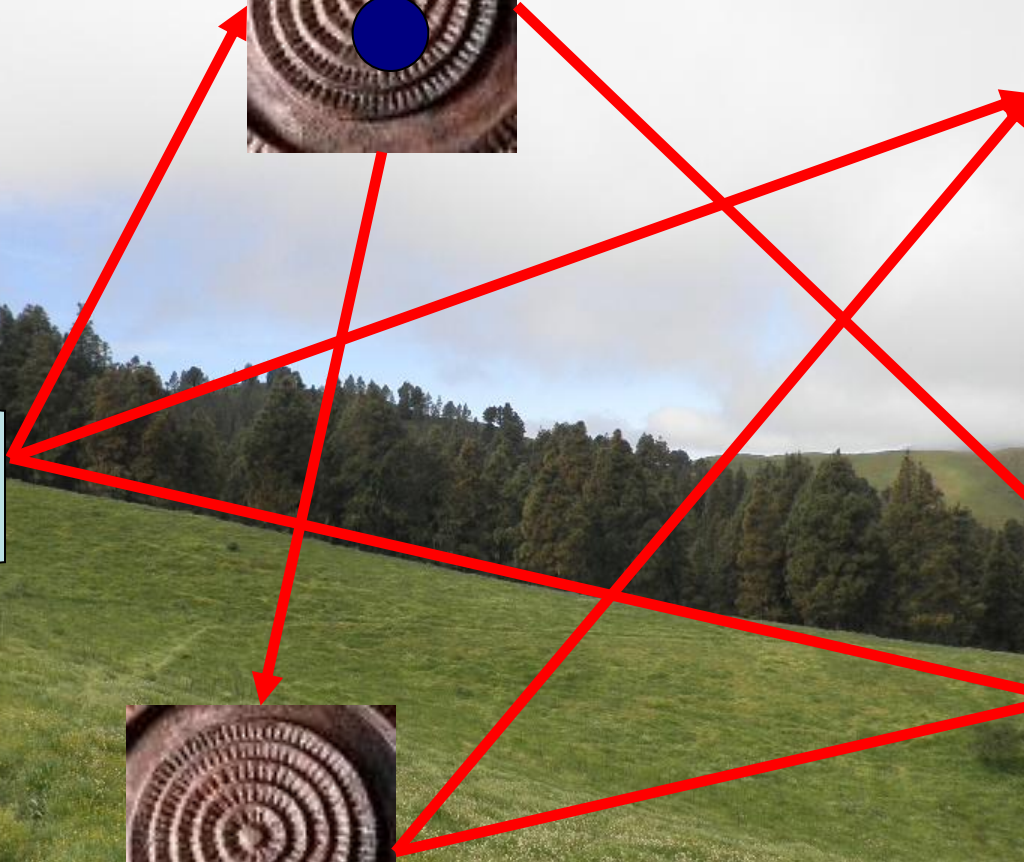
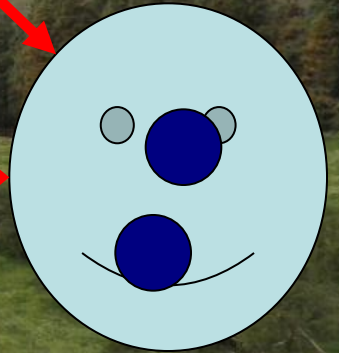
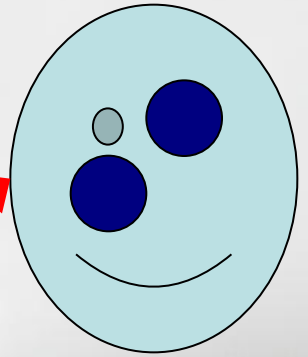
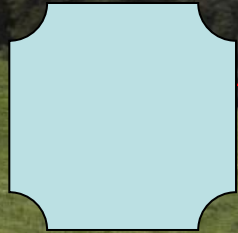
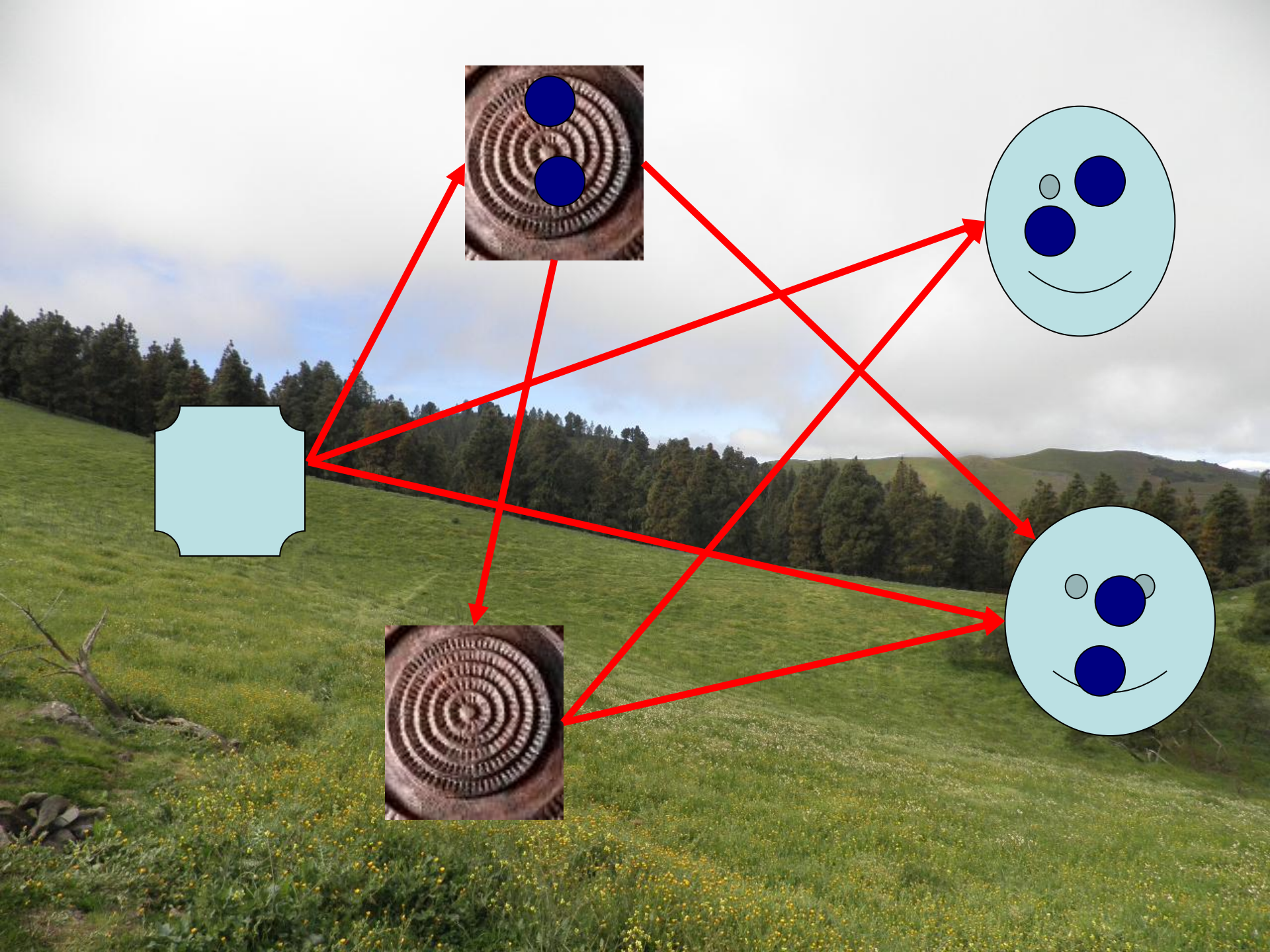


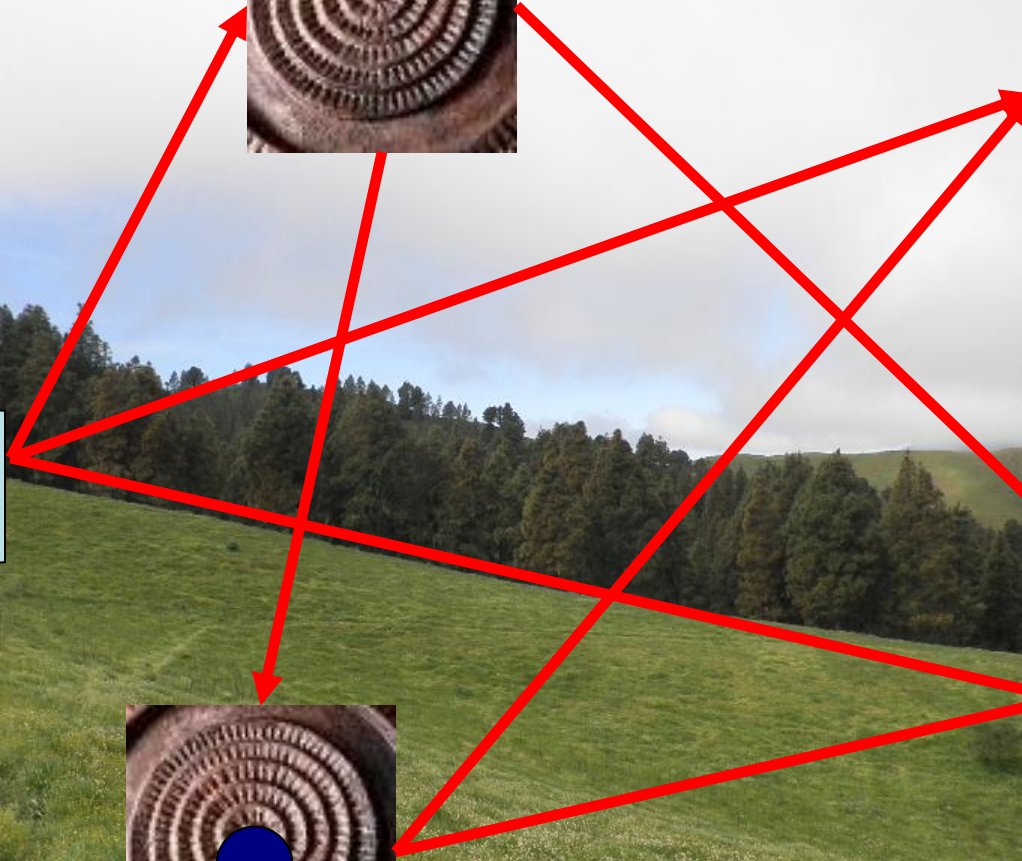
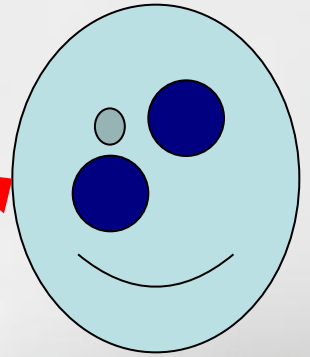
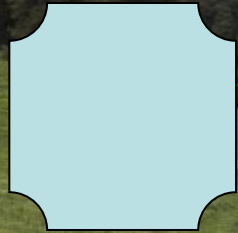
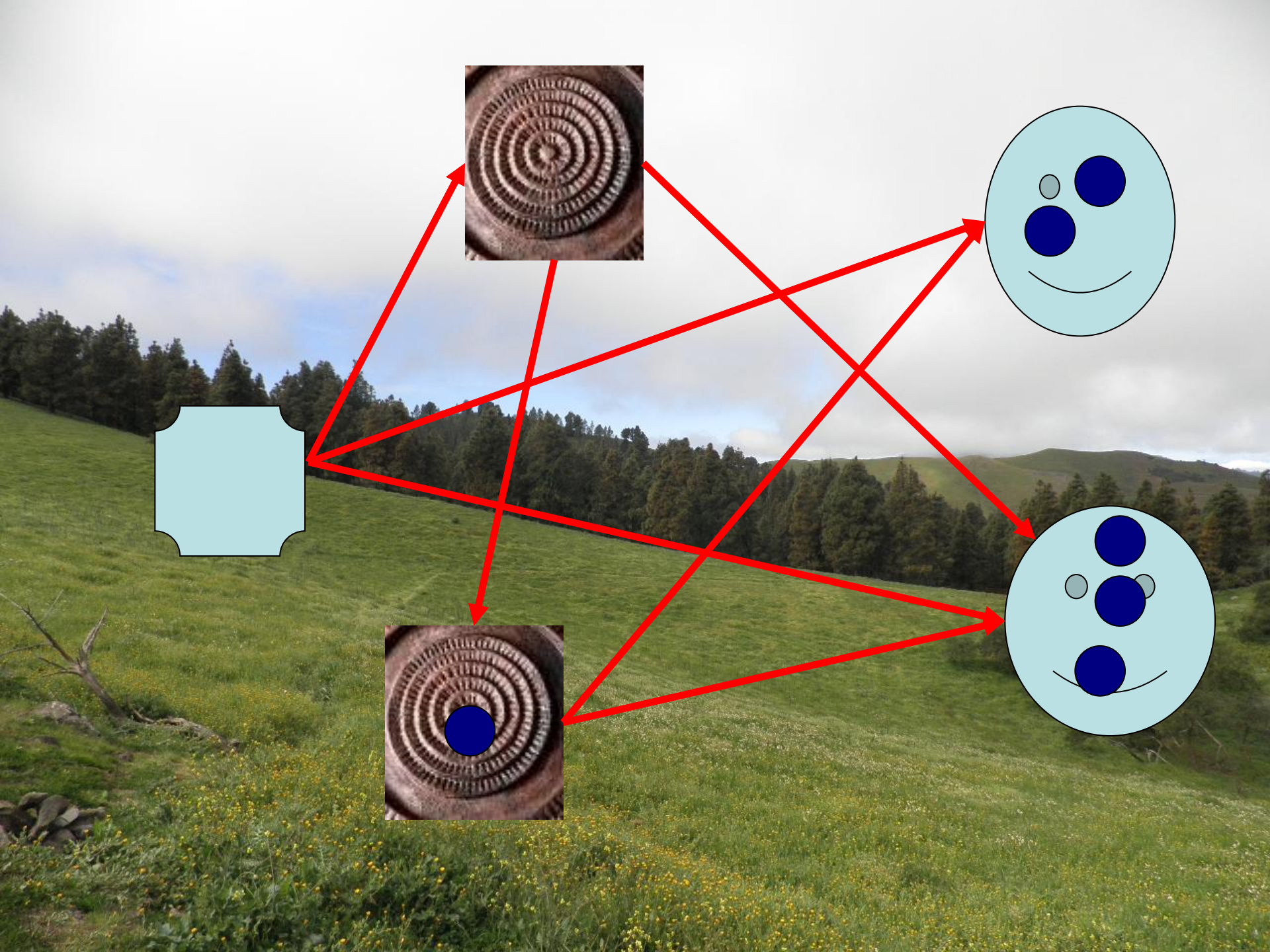
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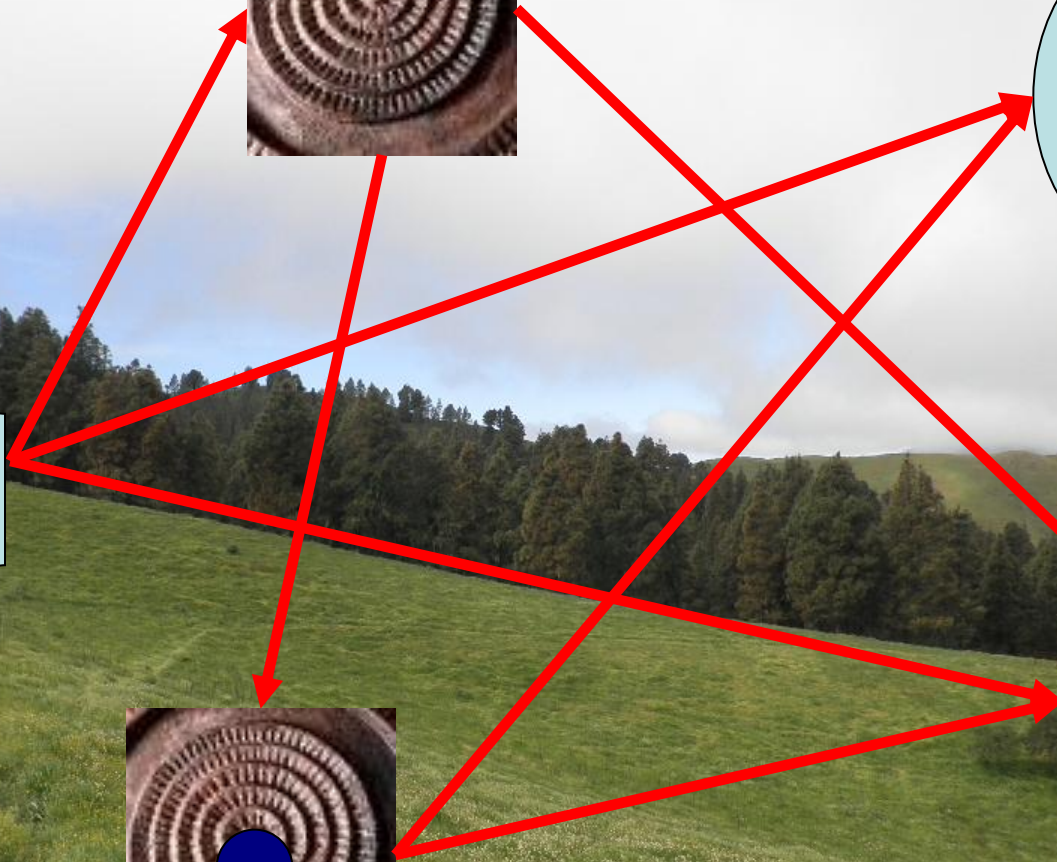
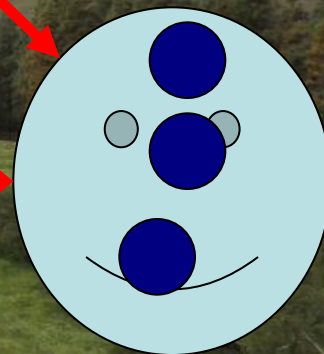
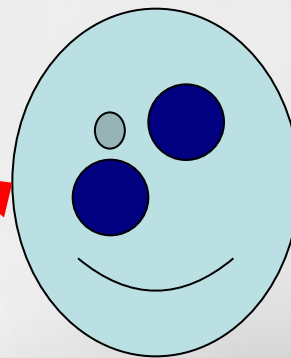
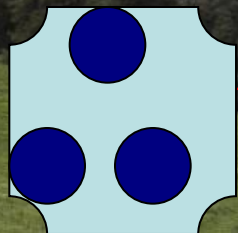
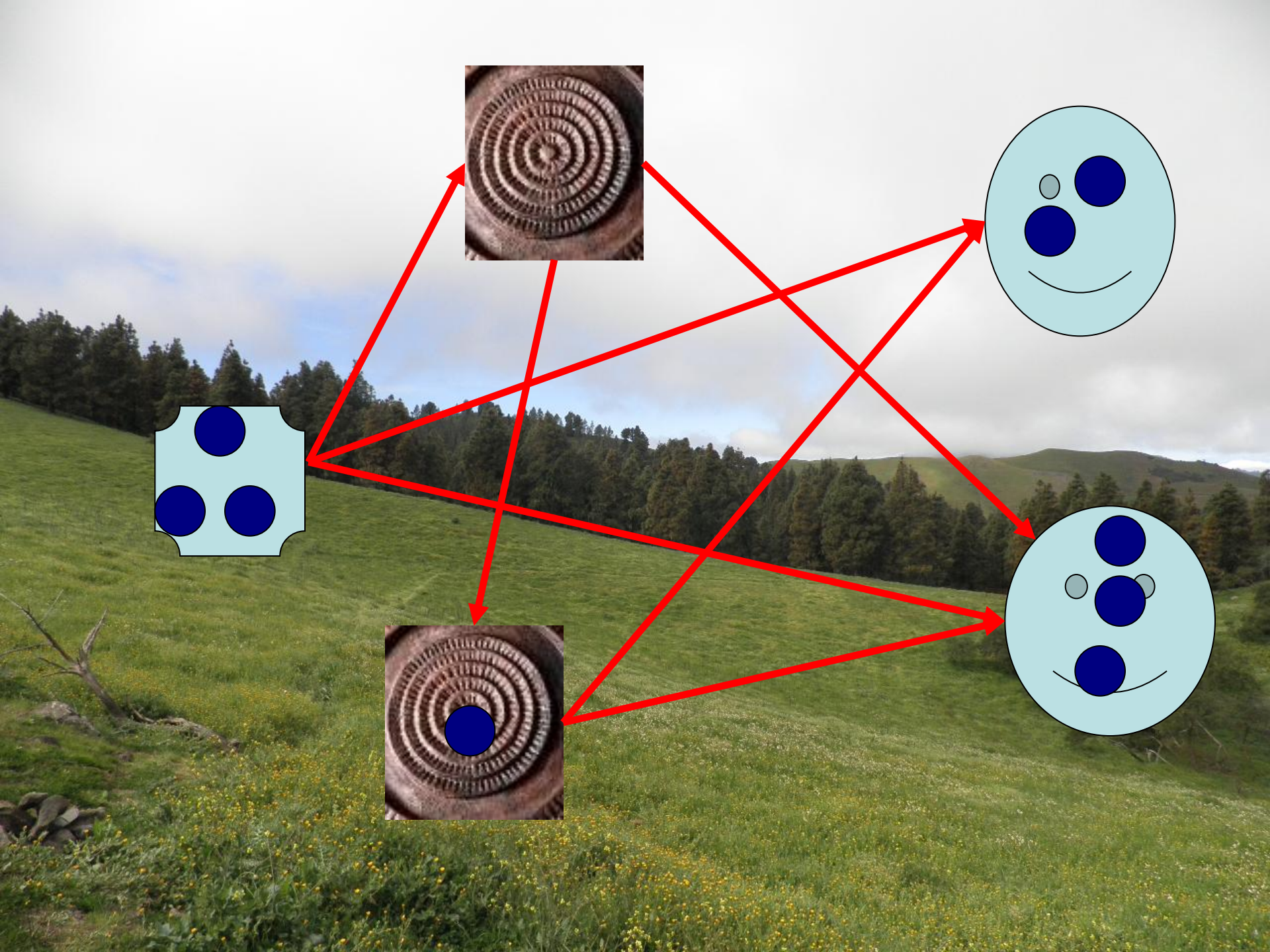


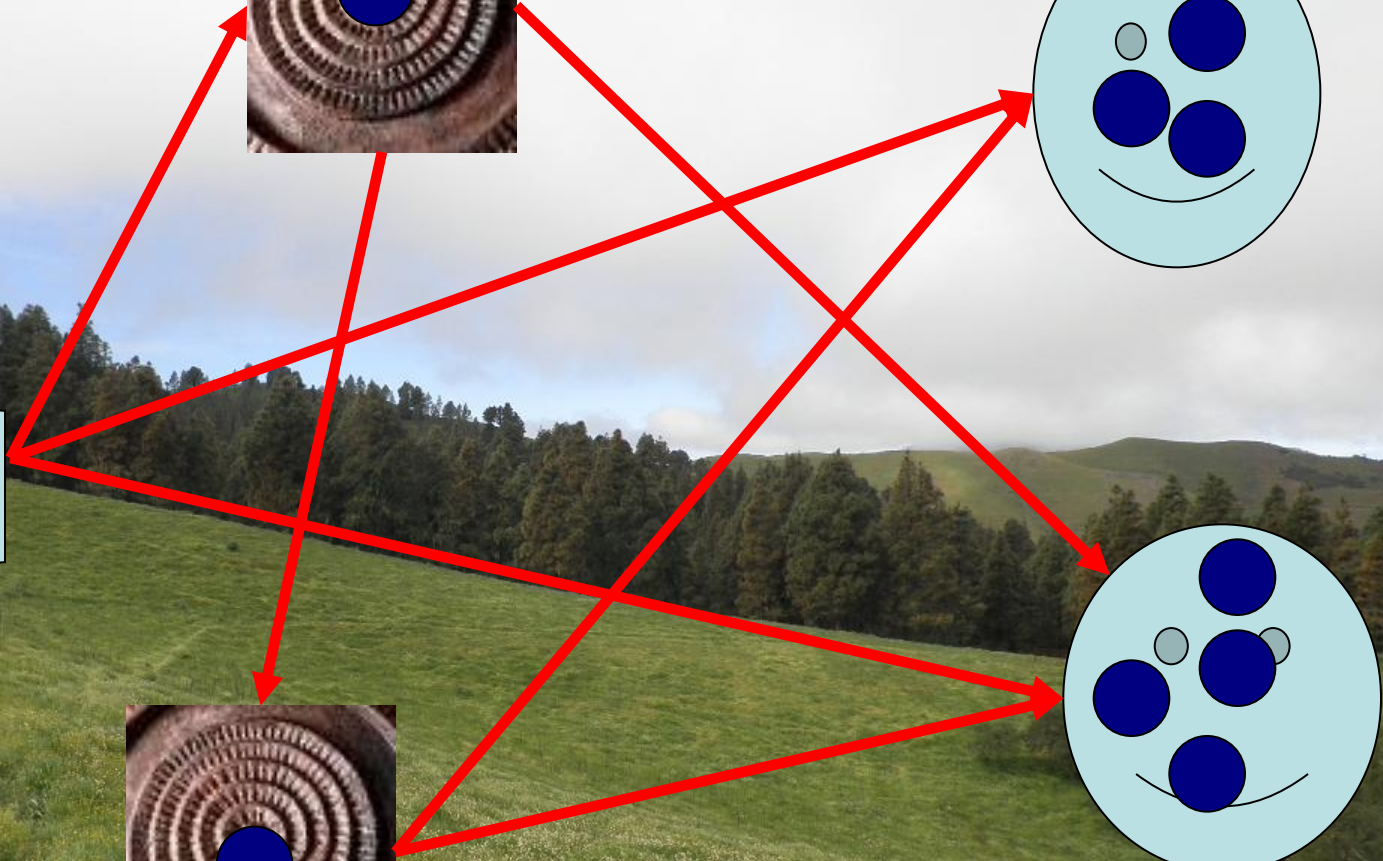
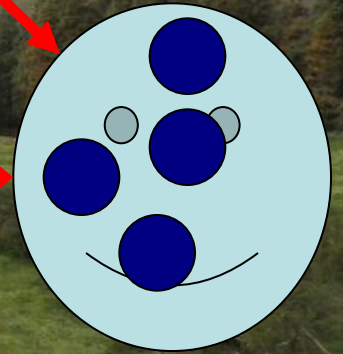
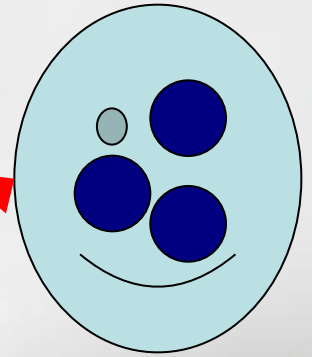
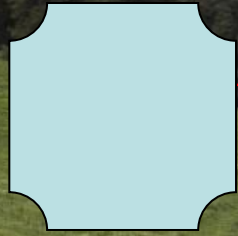
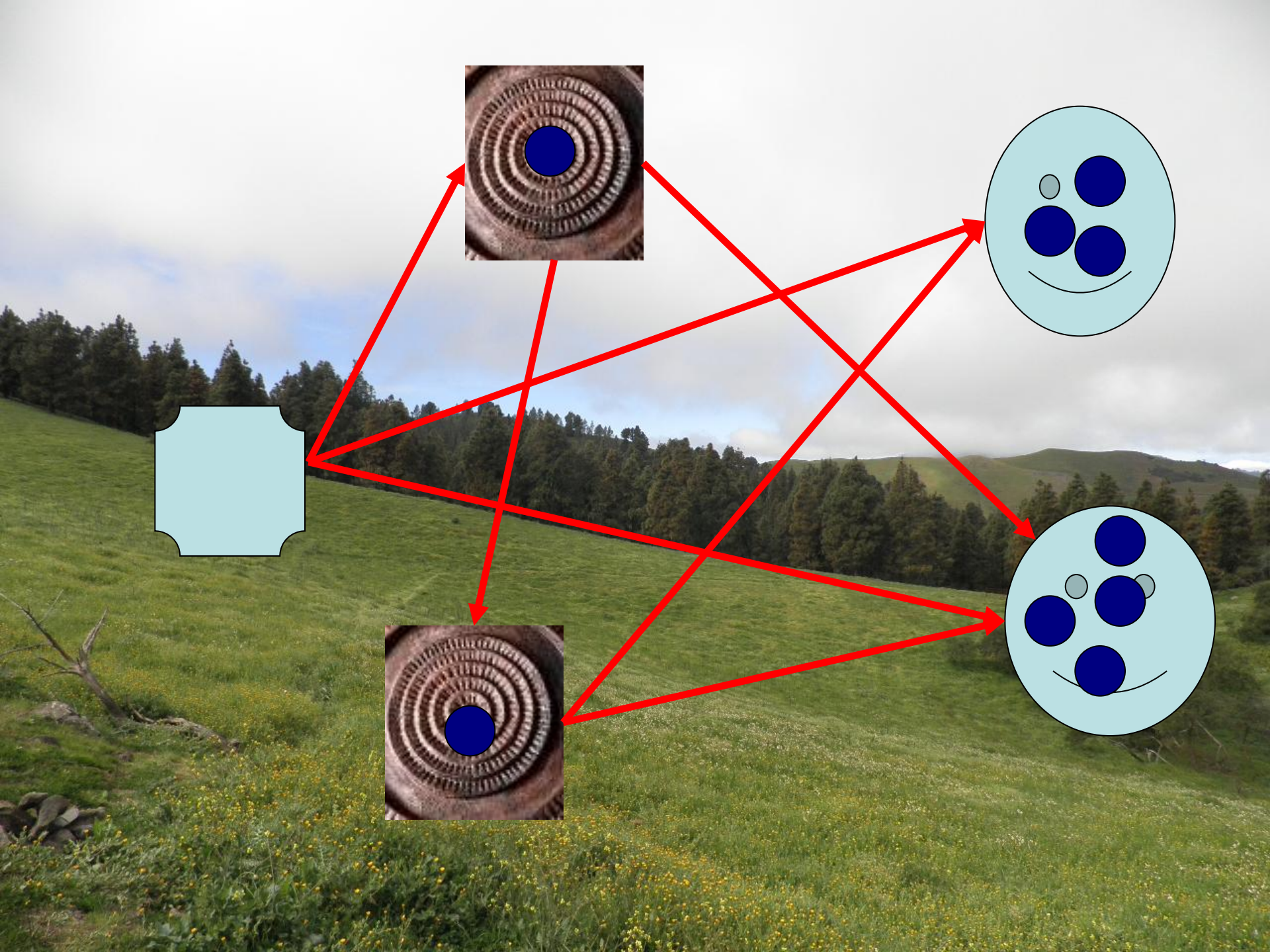


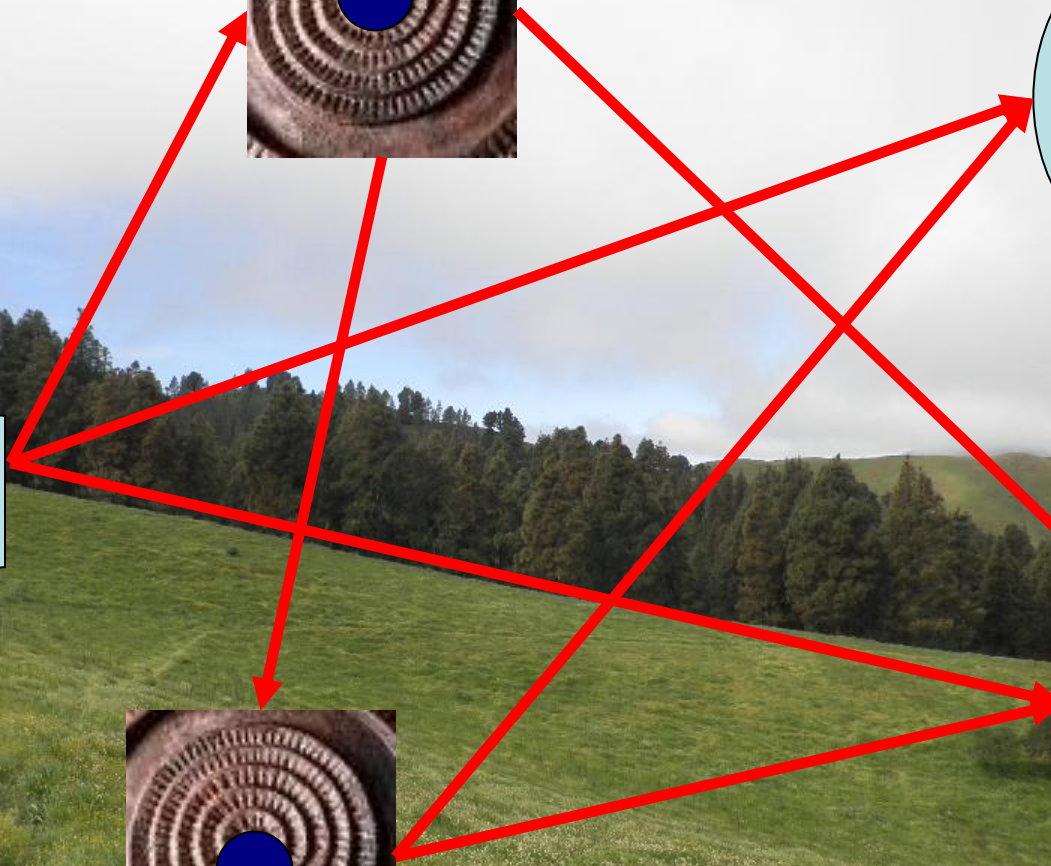
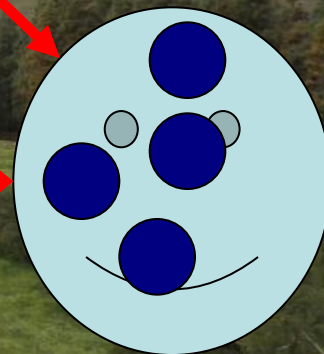
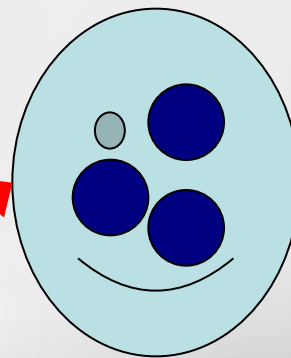
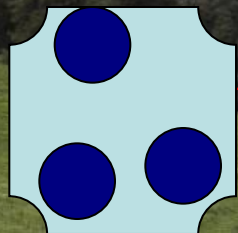
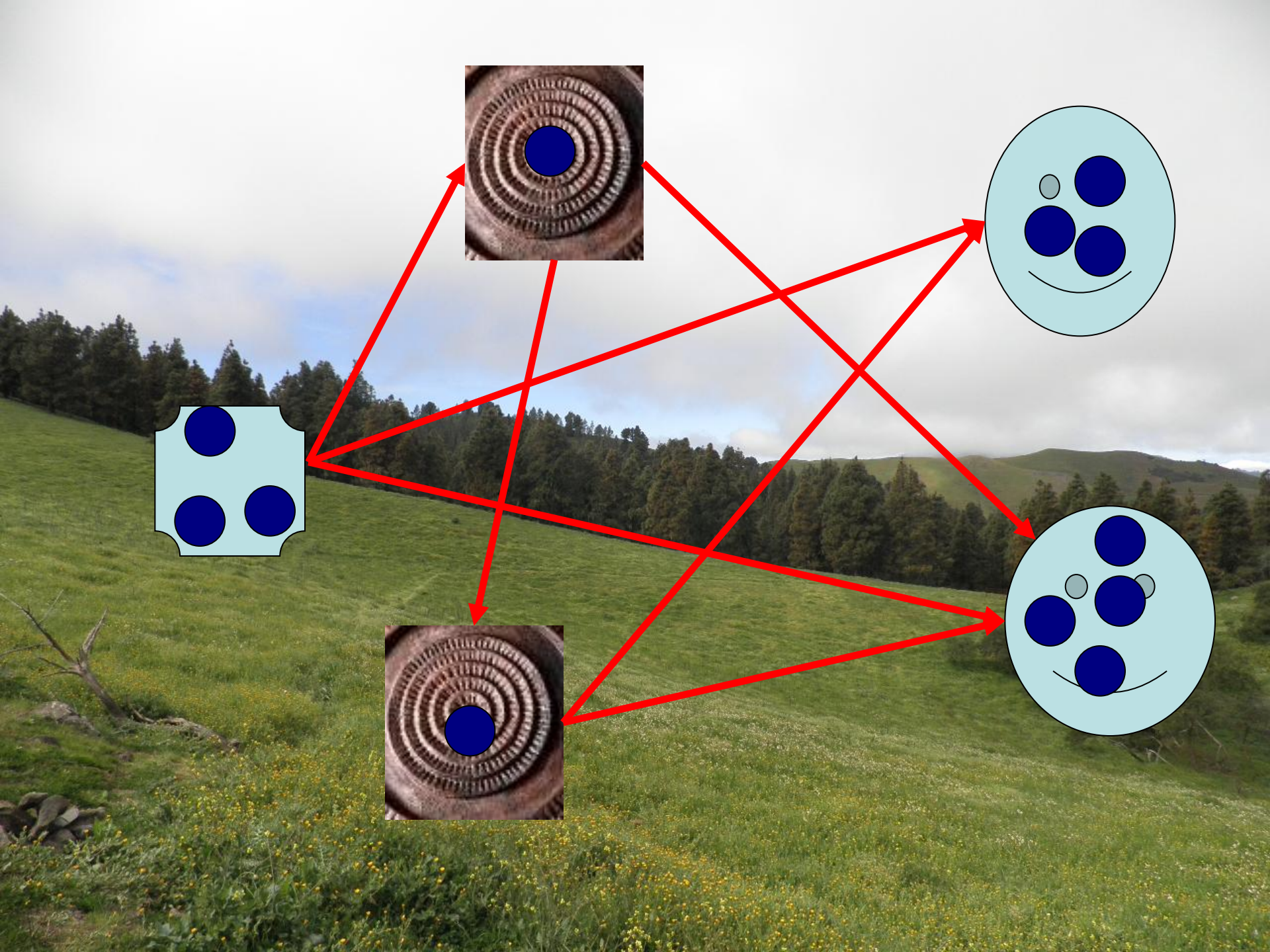


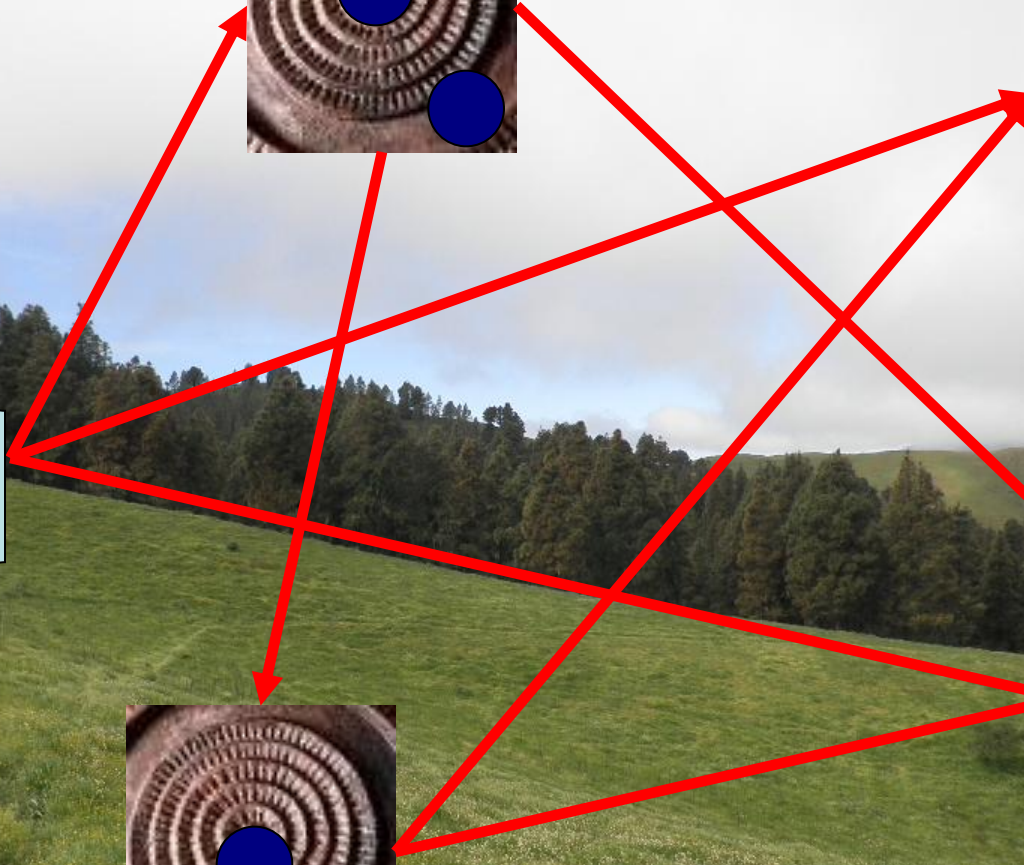
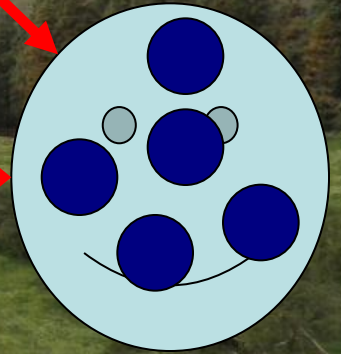
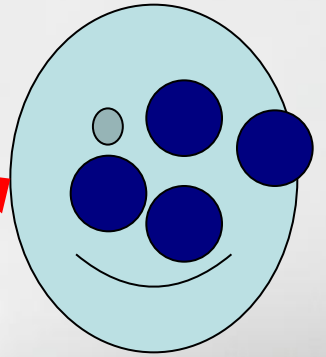
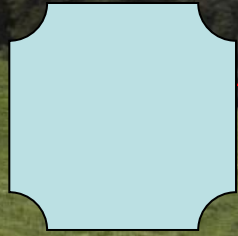
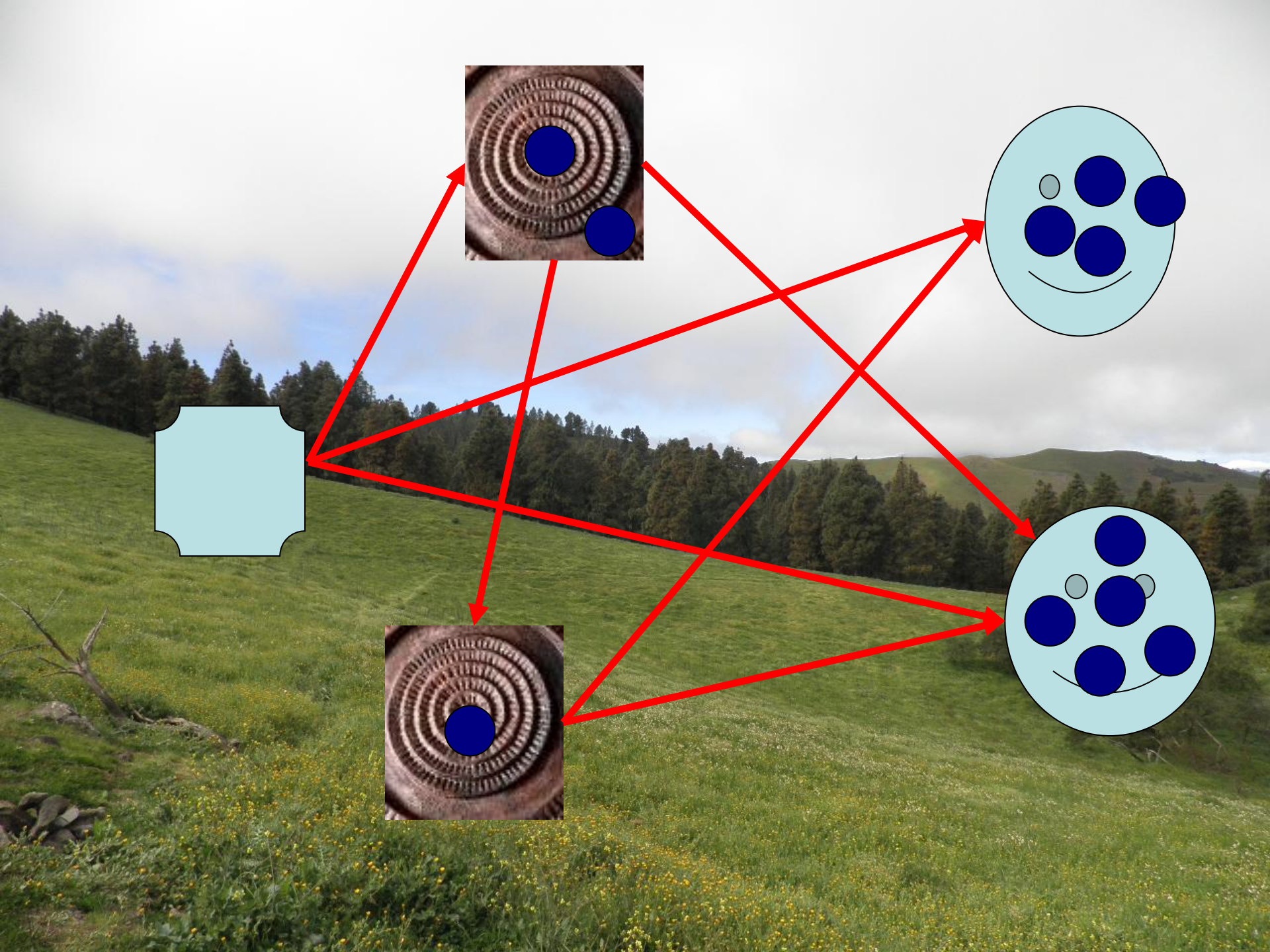


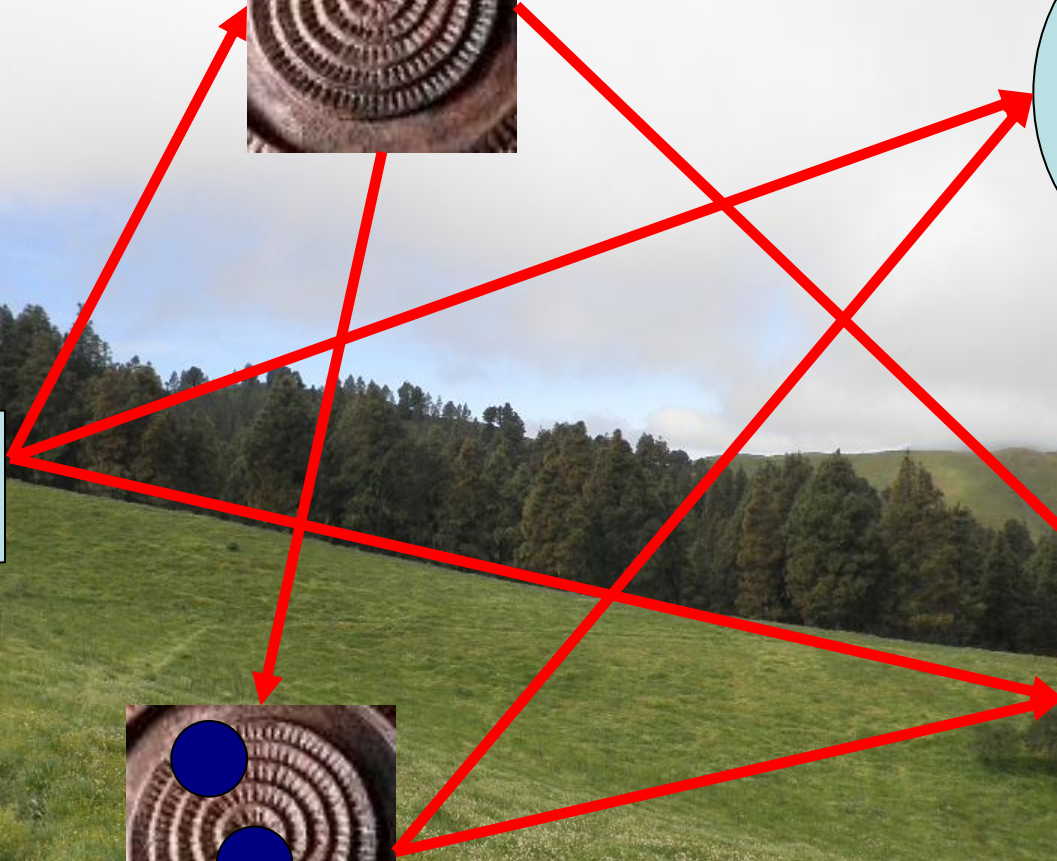
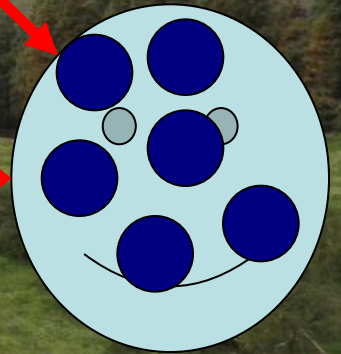
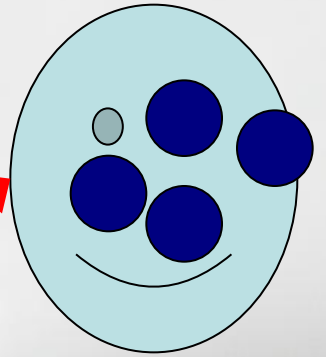
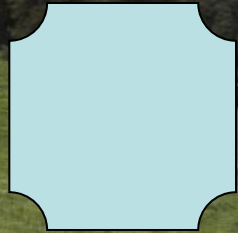
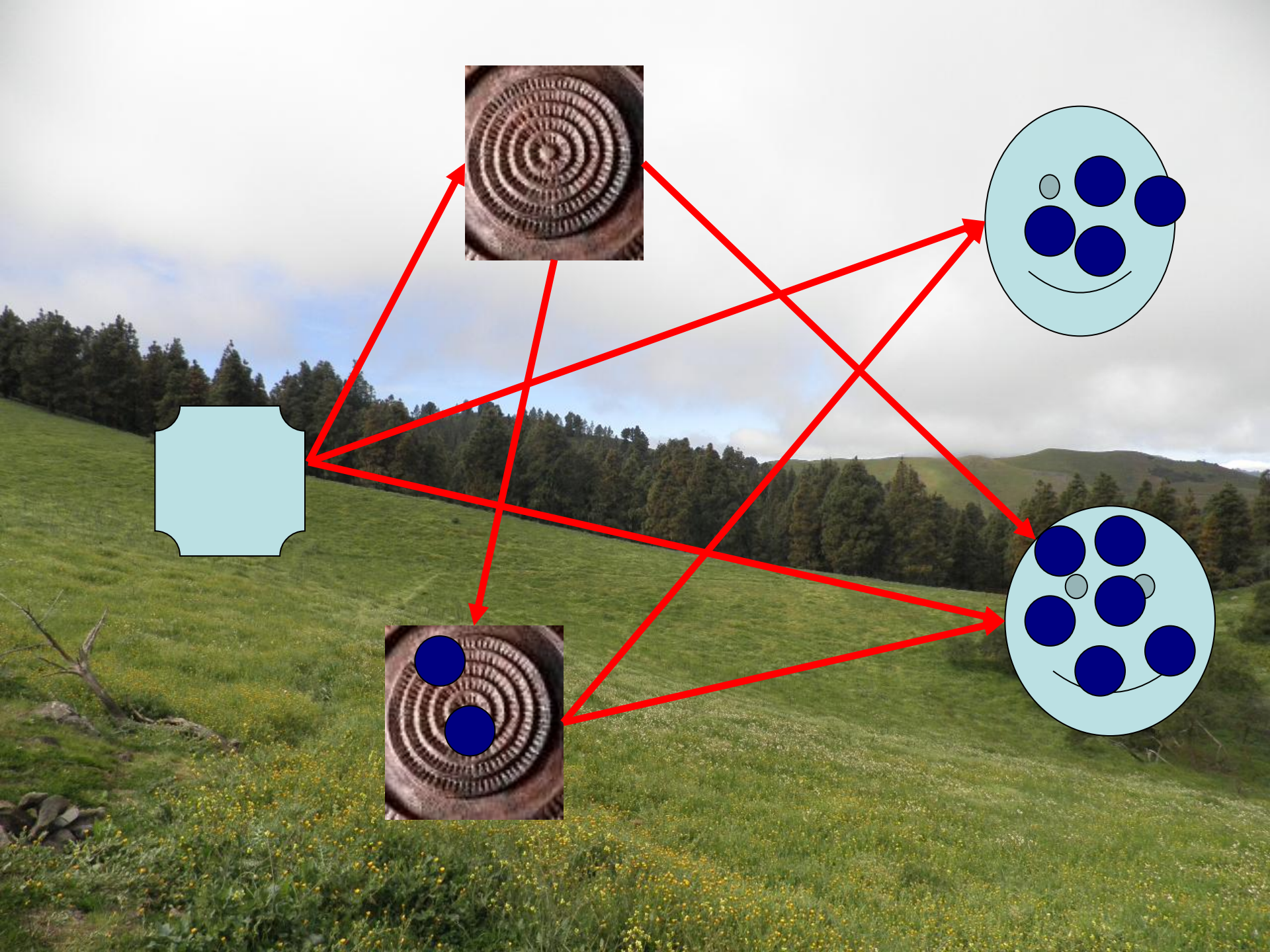


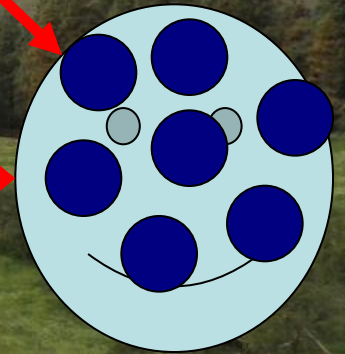
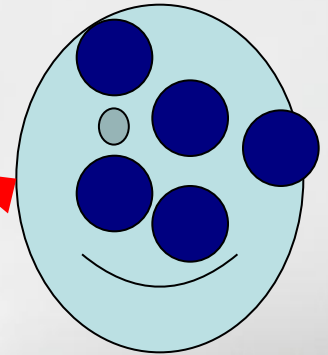
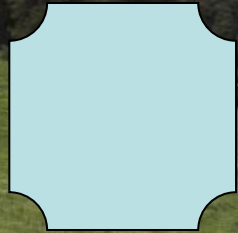






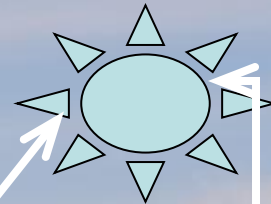
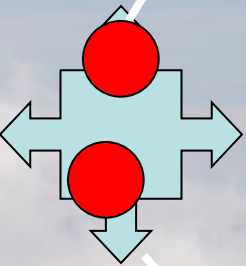


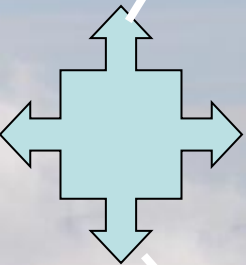
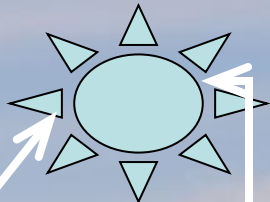


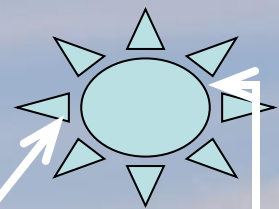
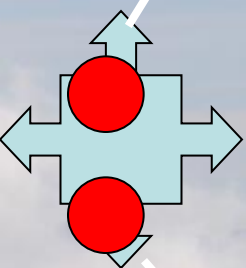


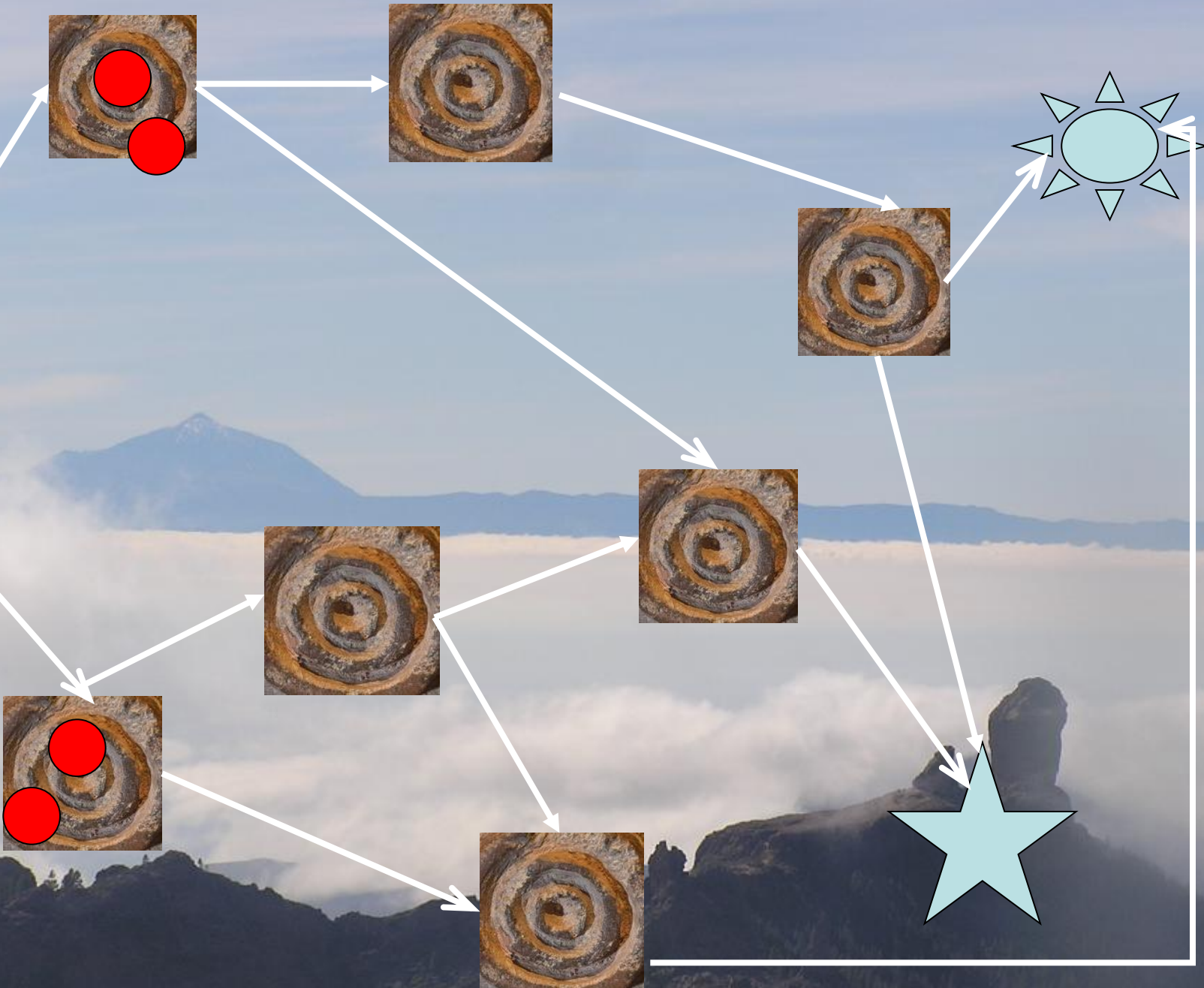
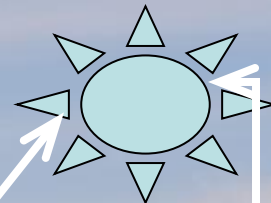
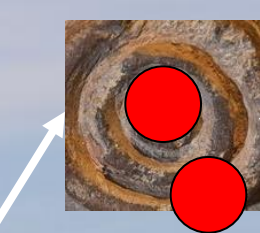
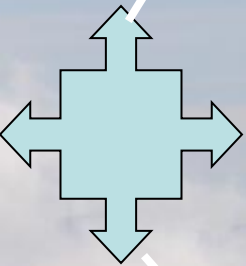
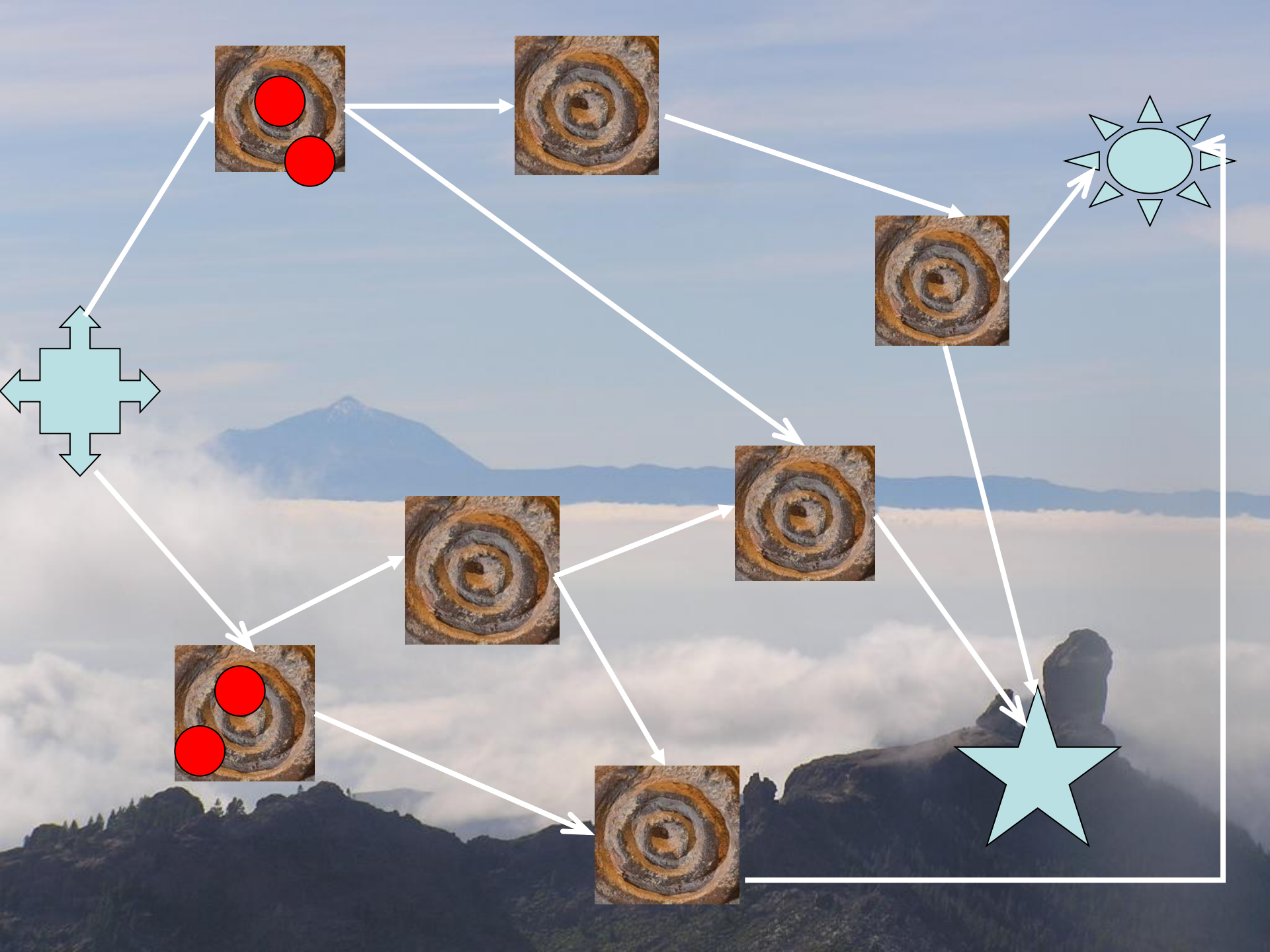
De 12 fichas, 5 llegan a la posición F1 y 7 a la F2

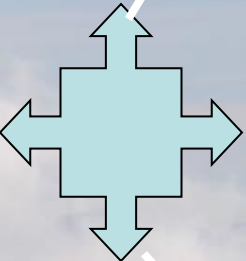
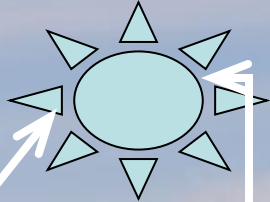
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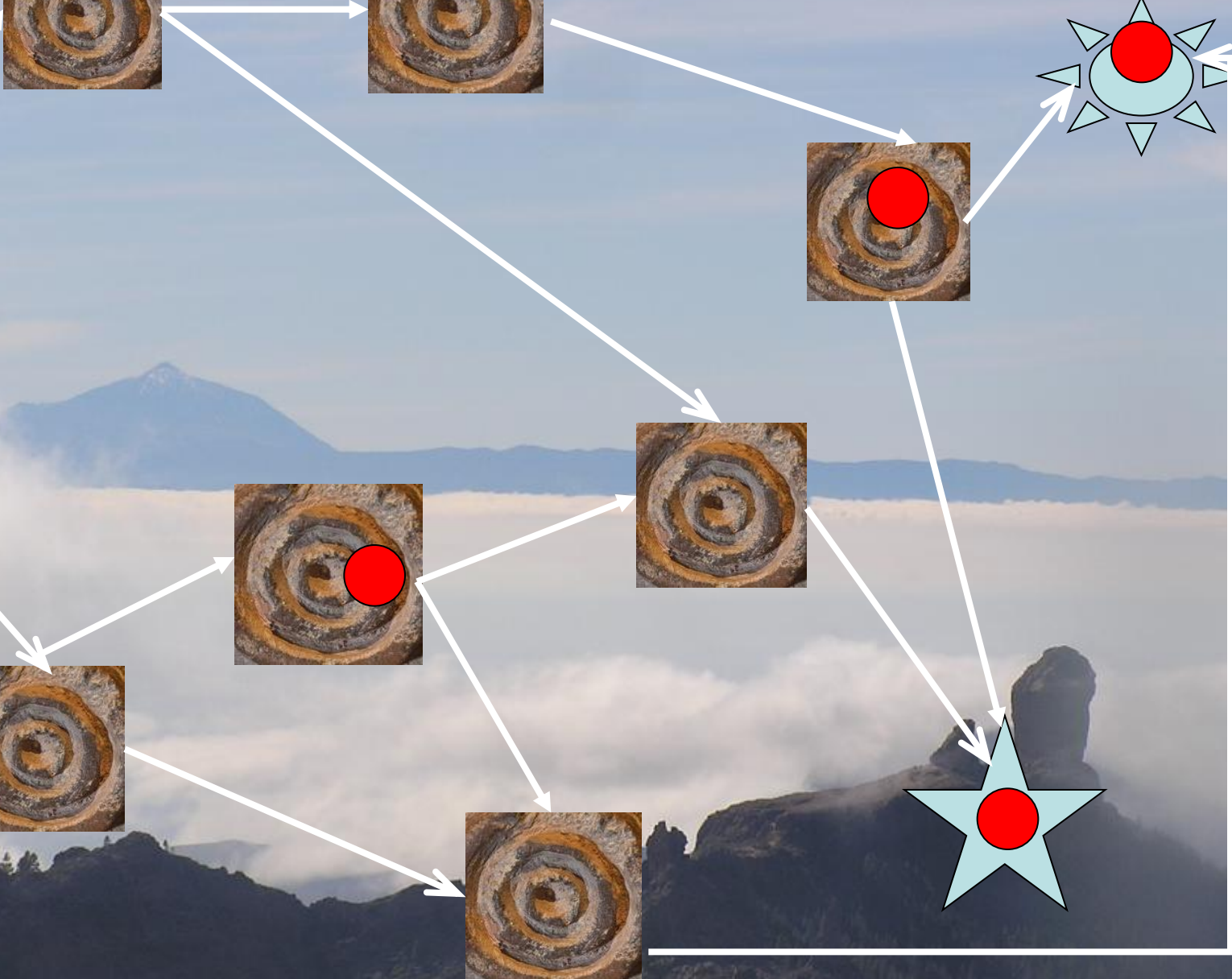
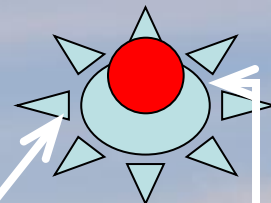
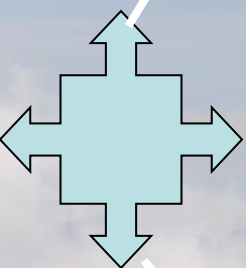
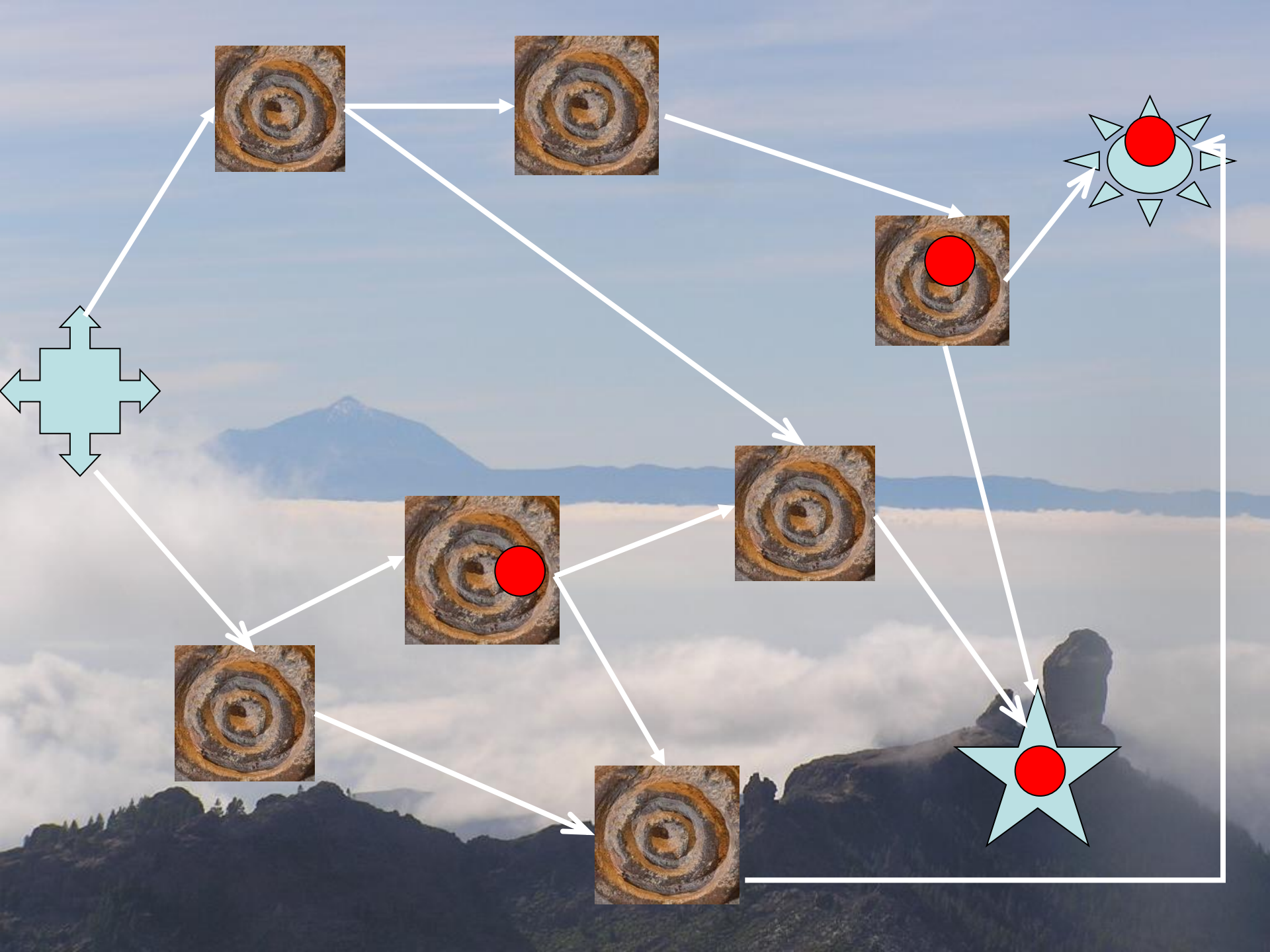


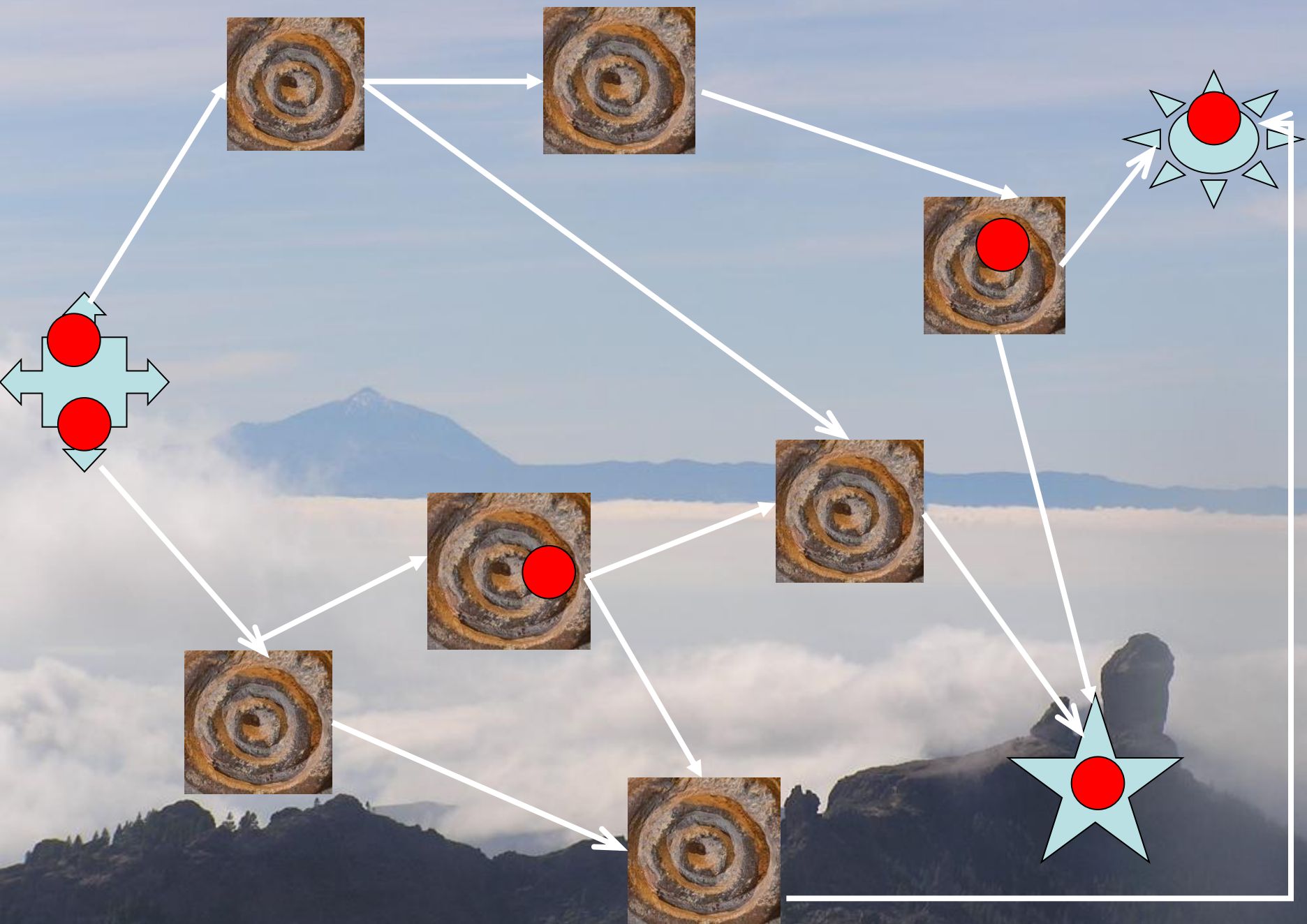
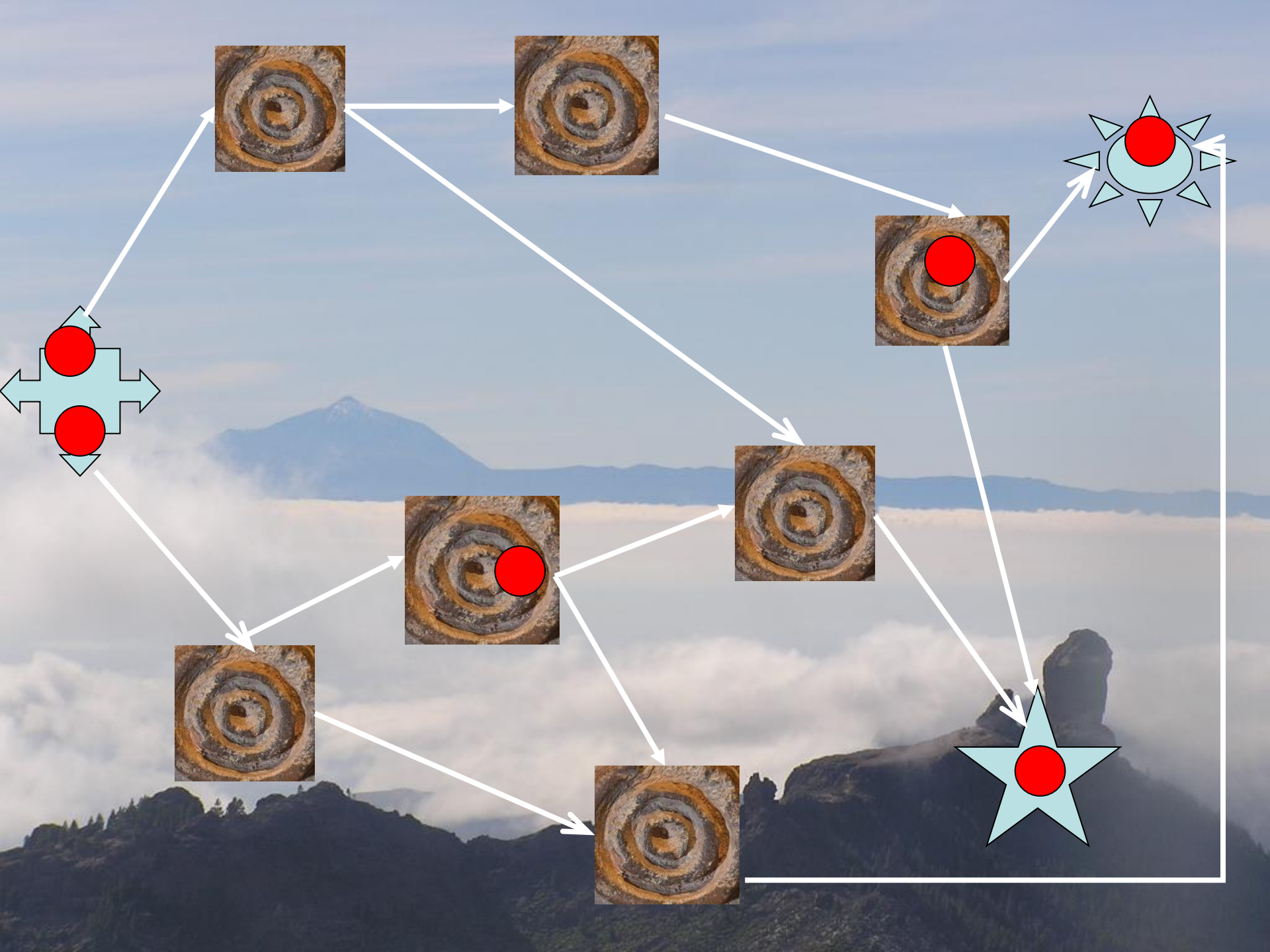


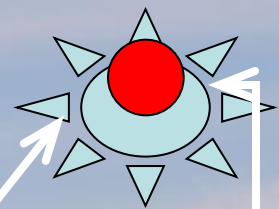
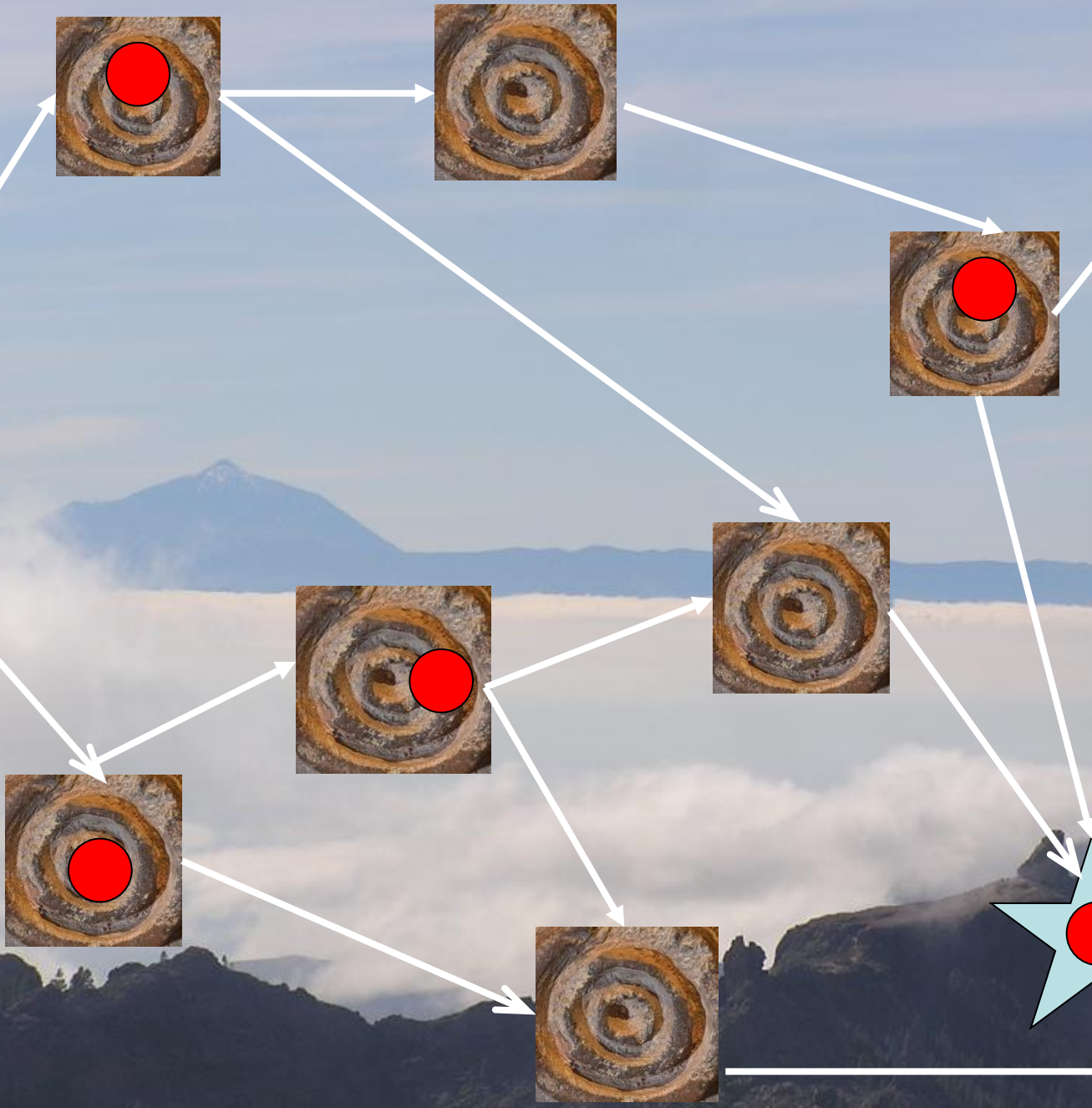
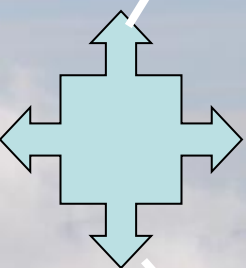
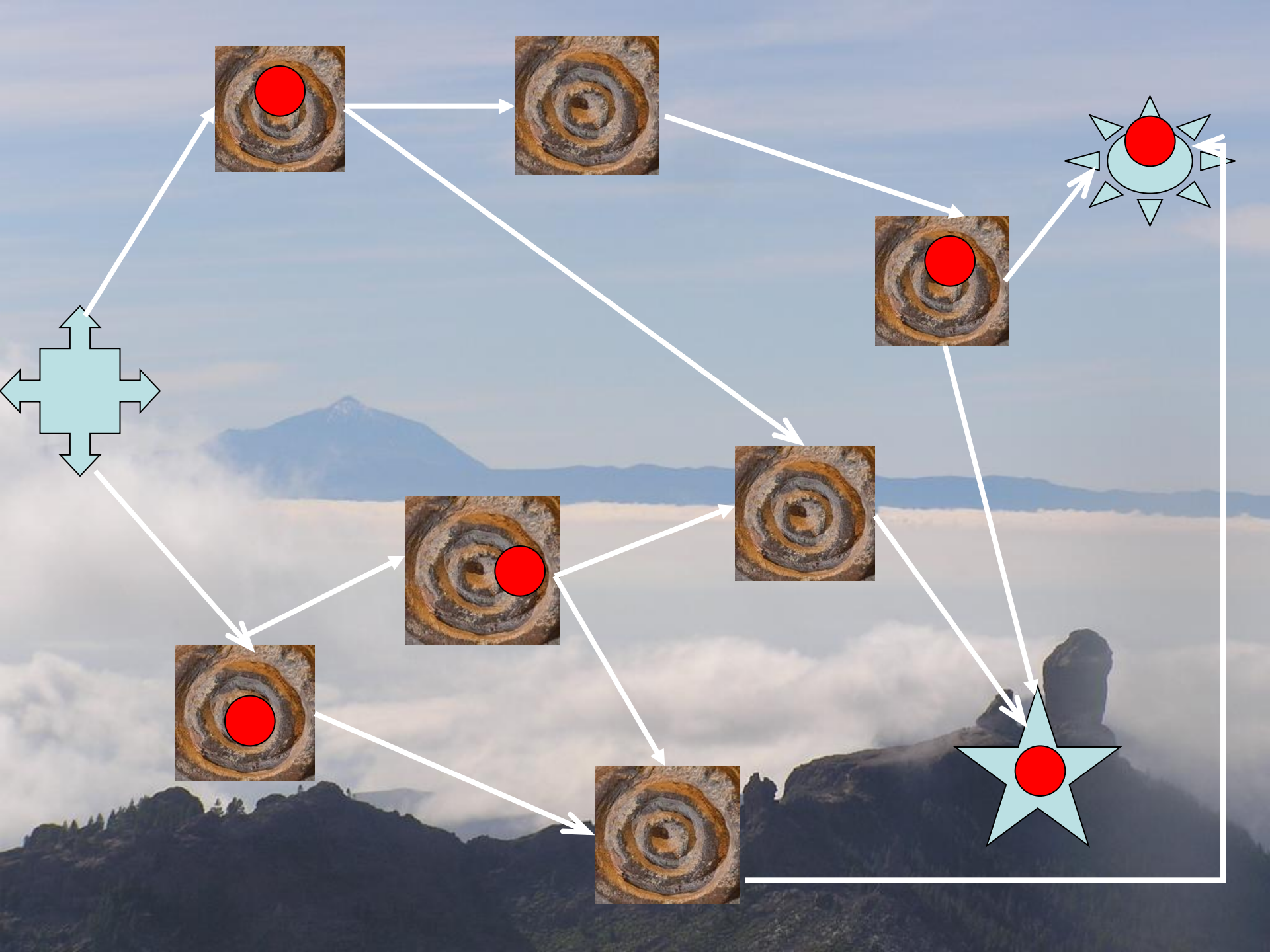


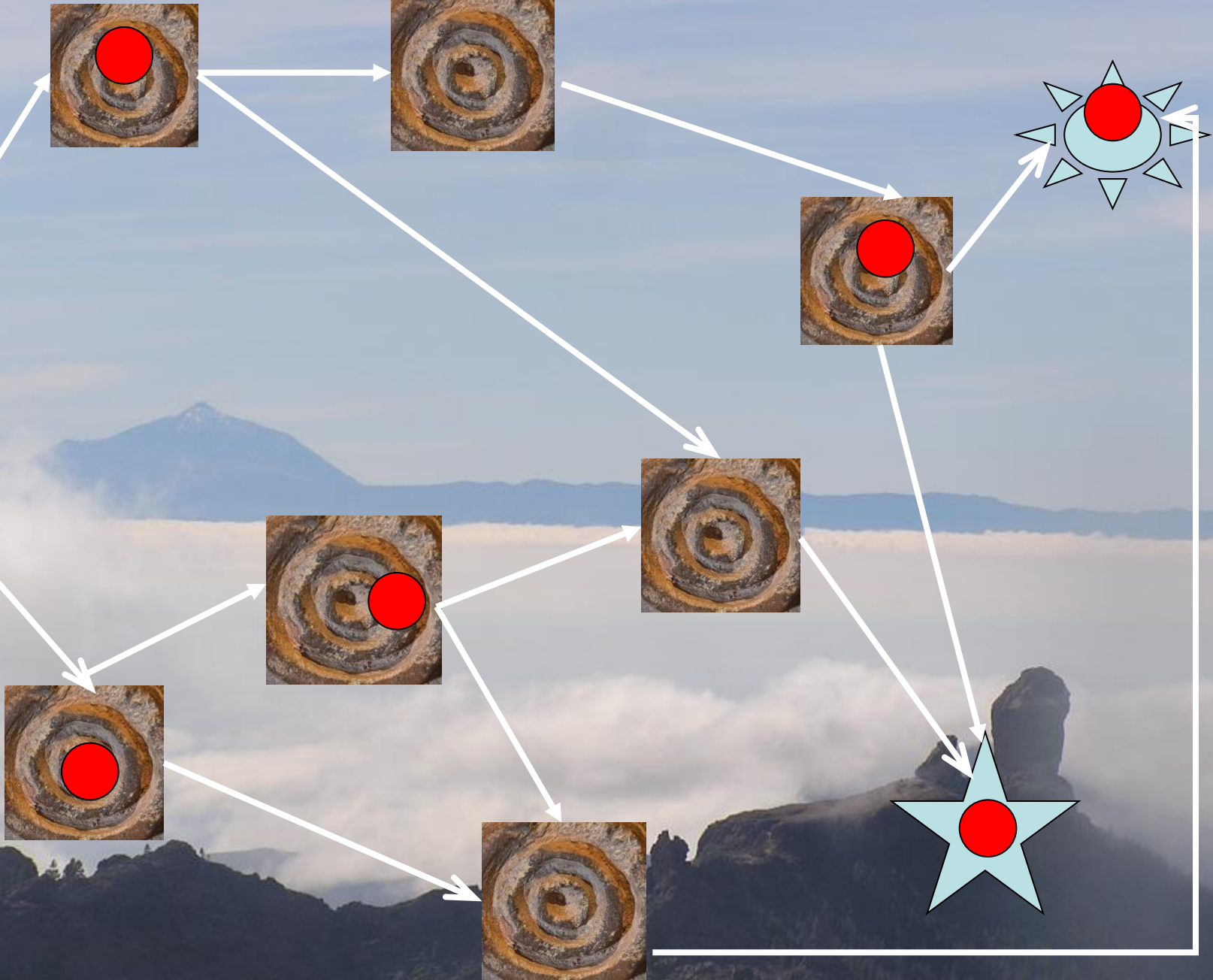
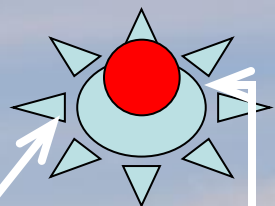
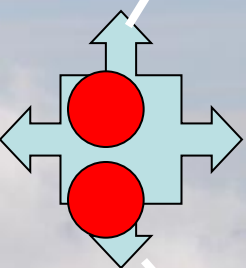
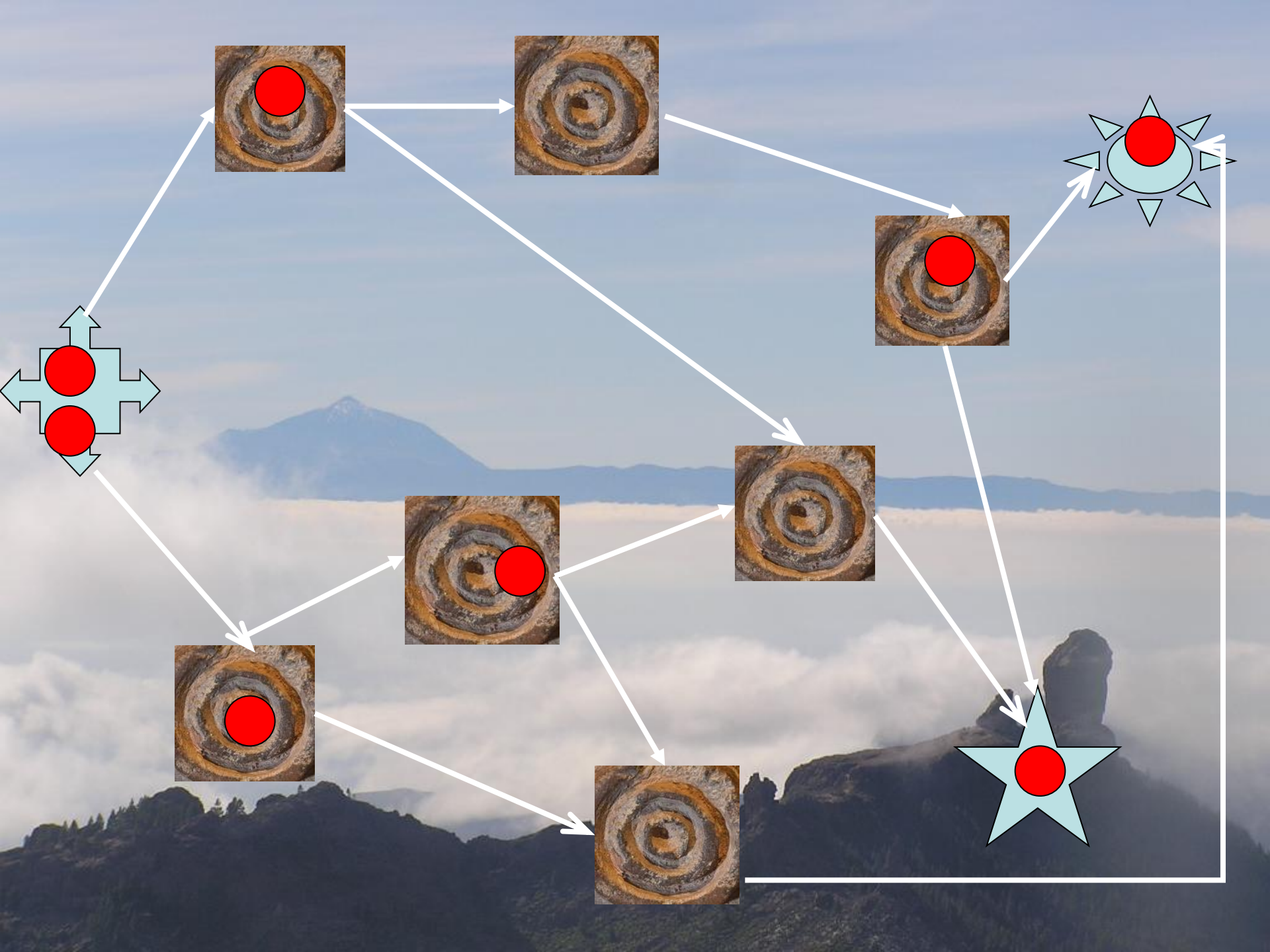


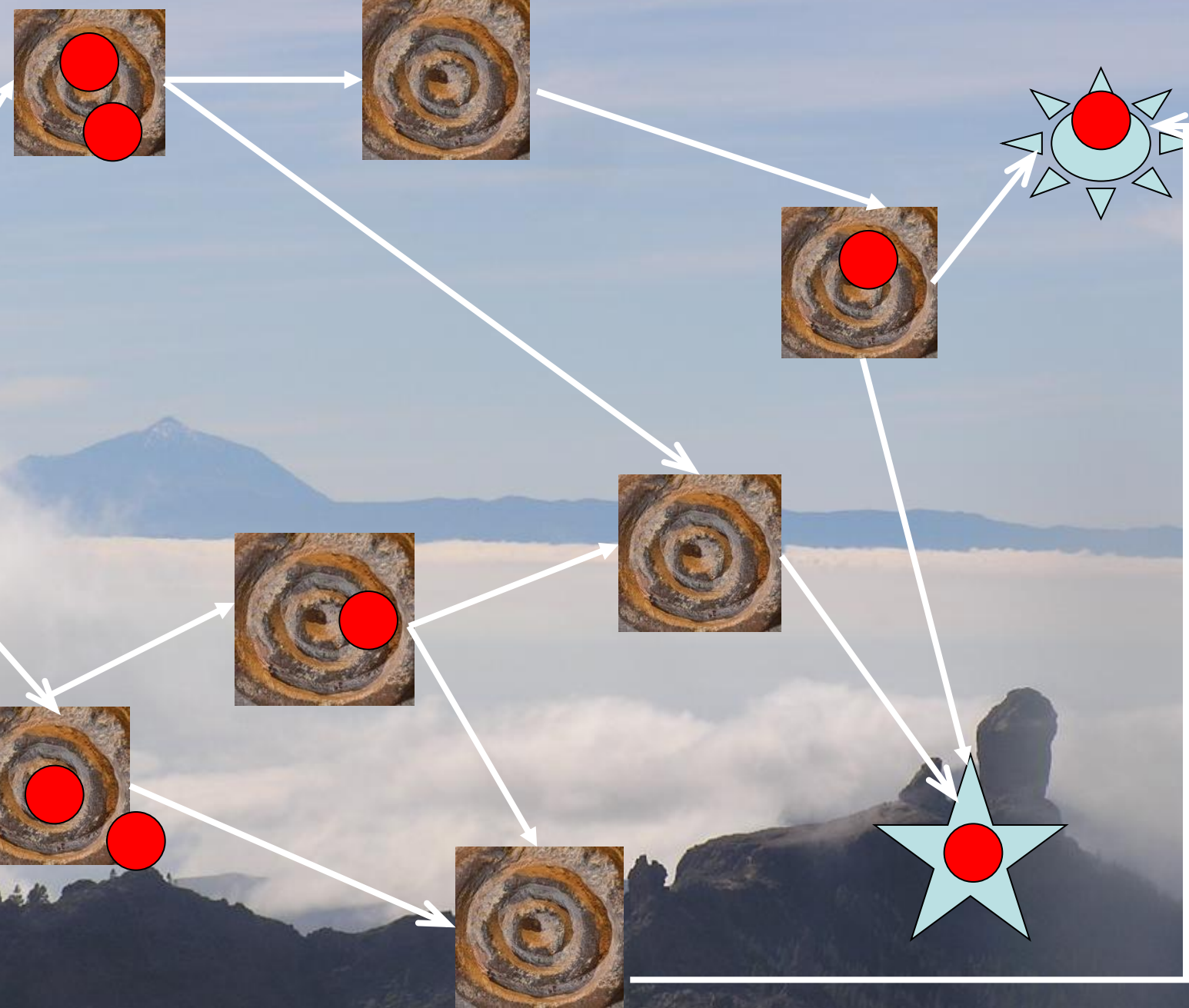
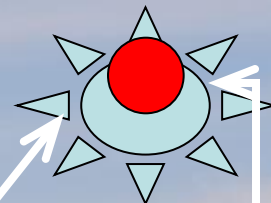
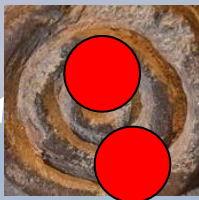
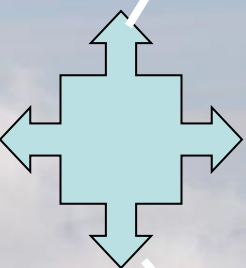
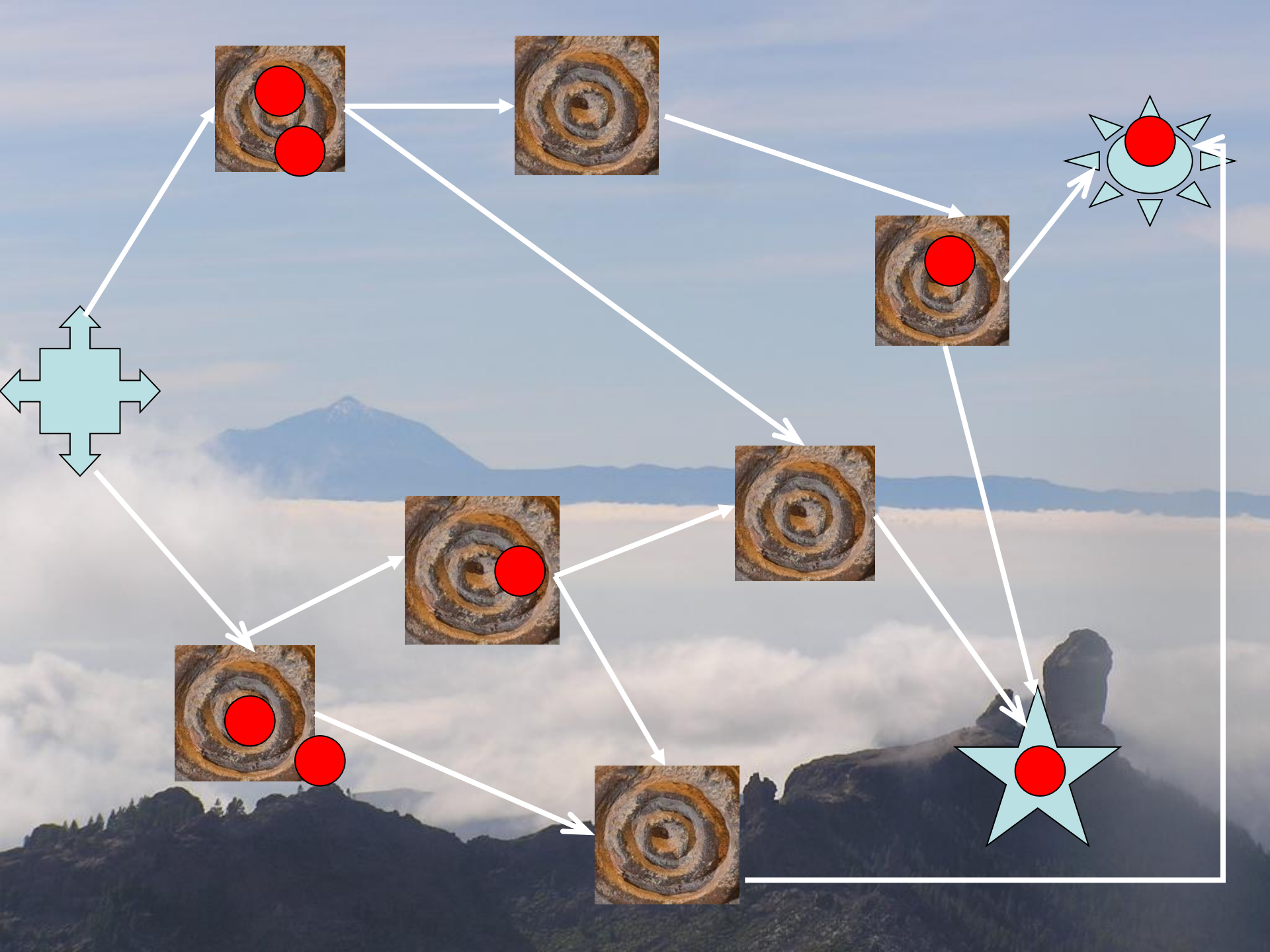


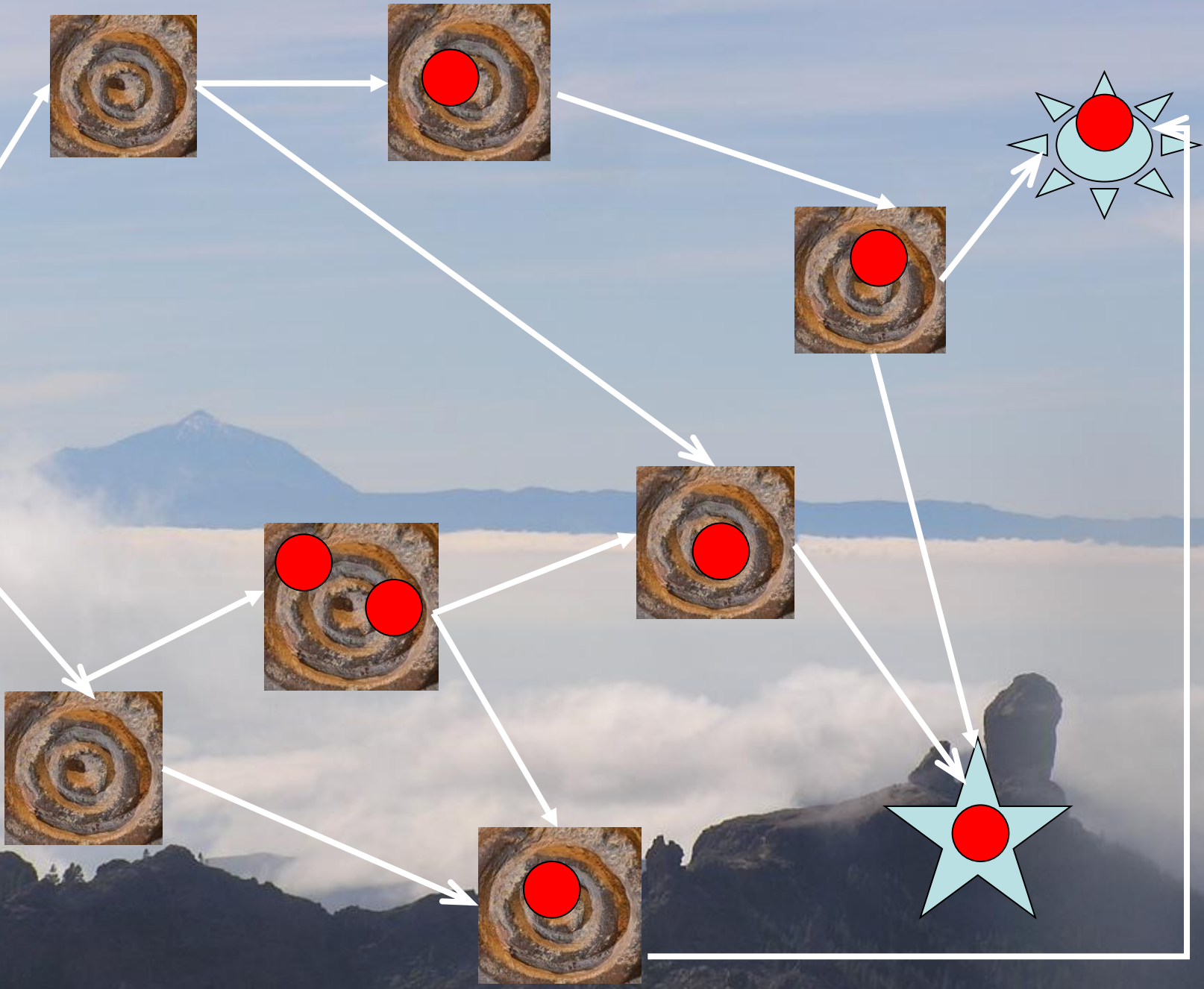
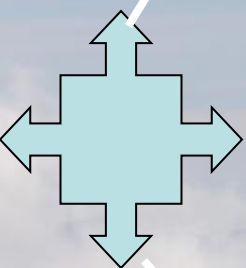
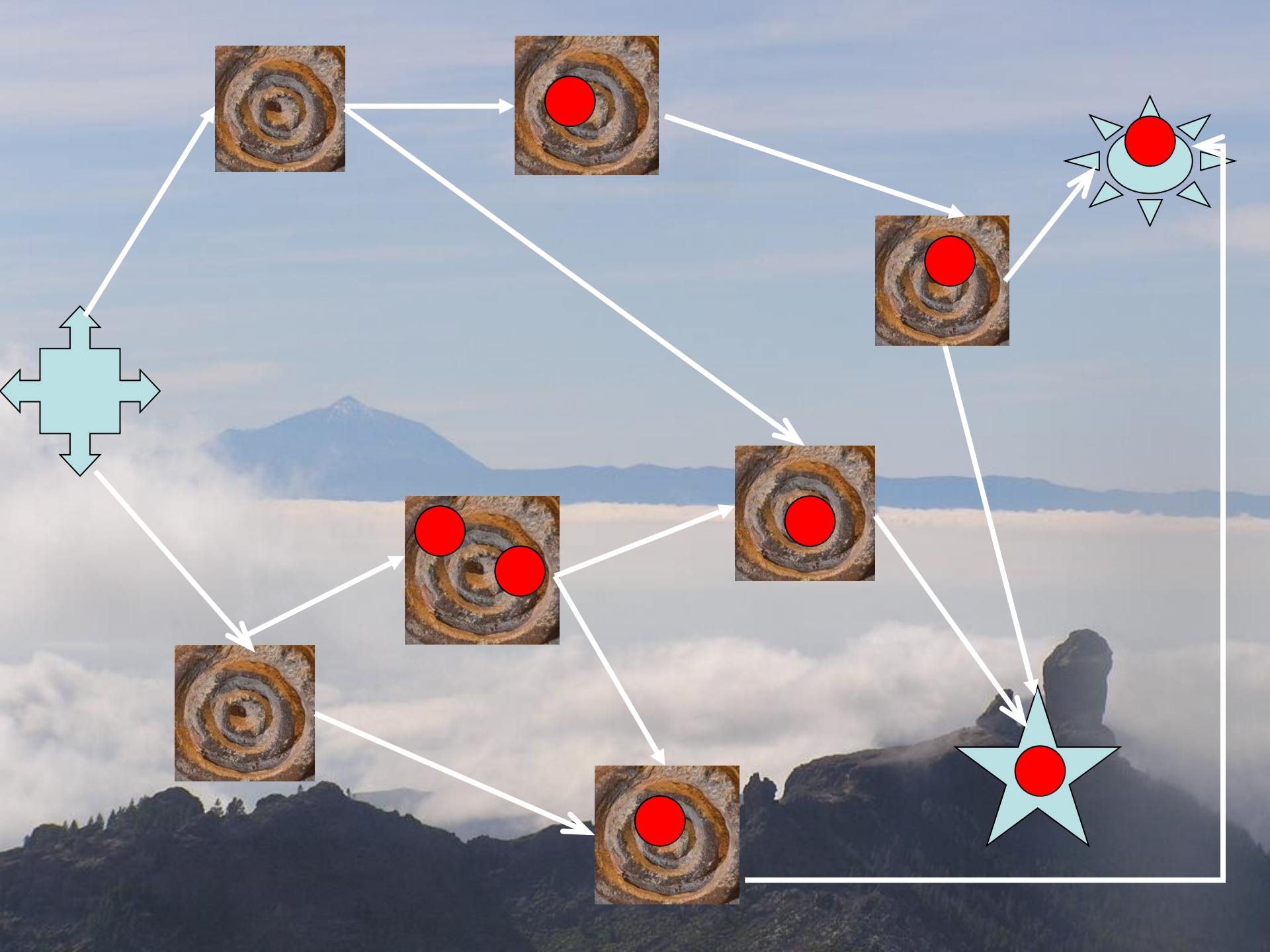


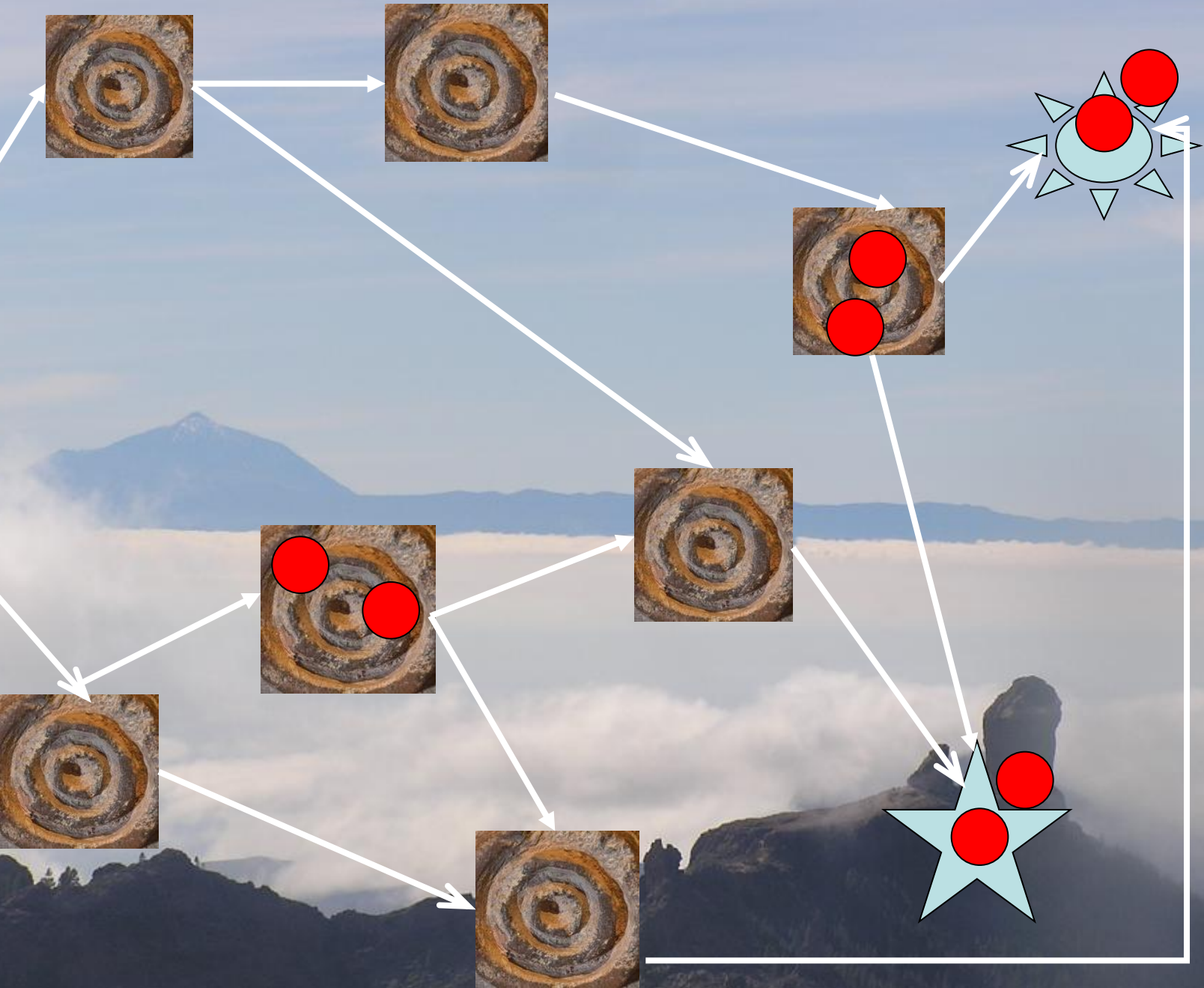
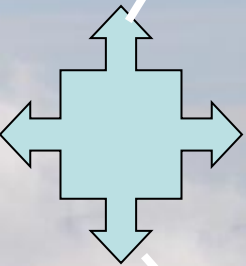
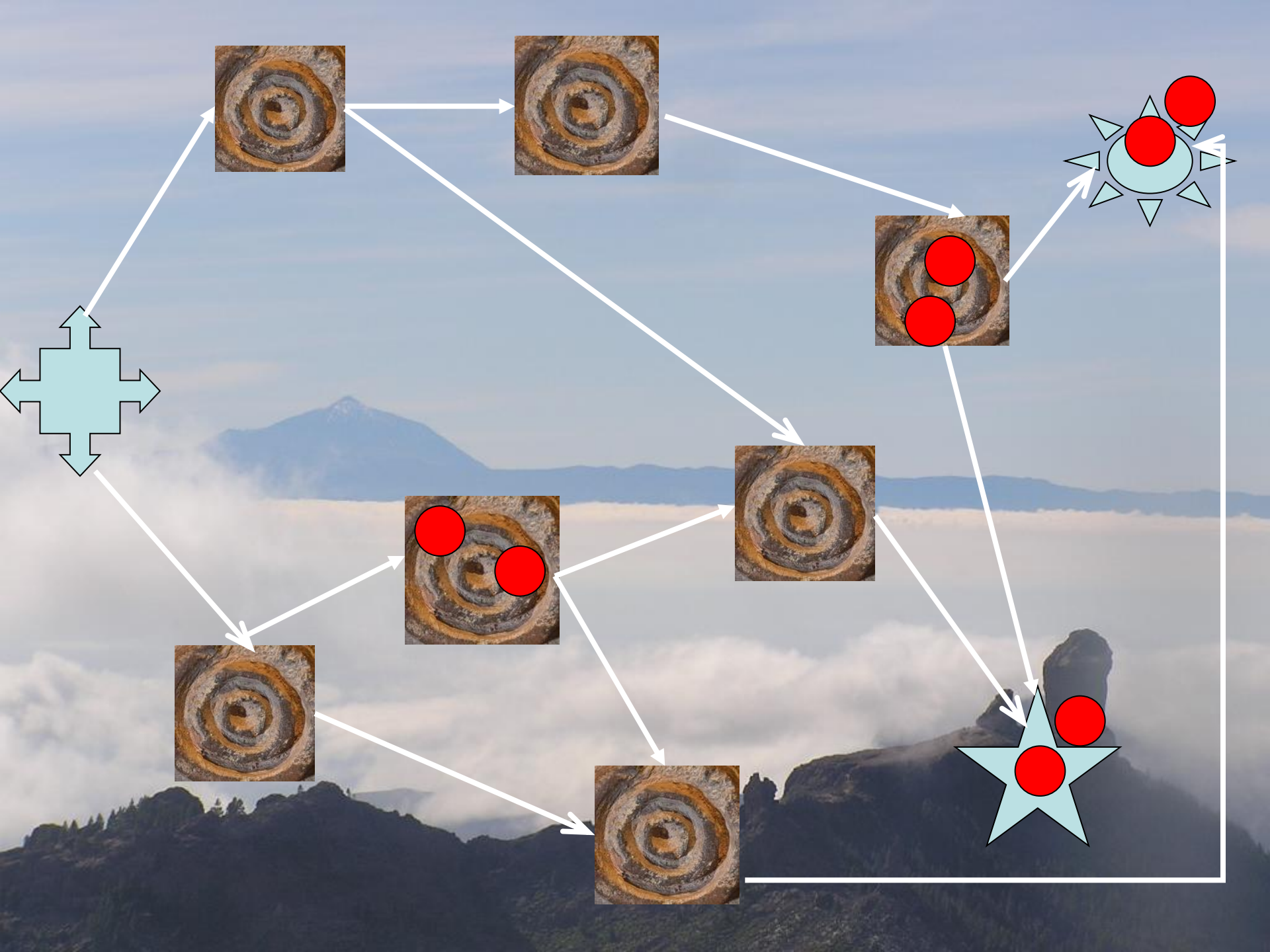


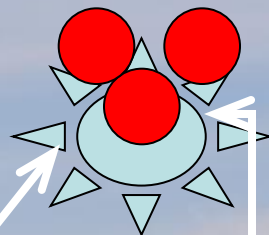
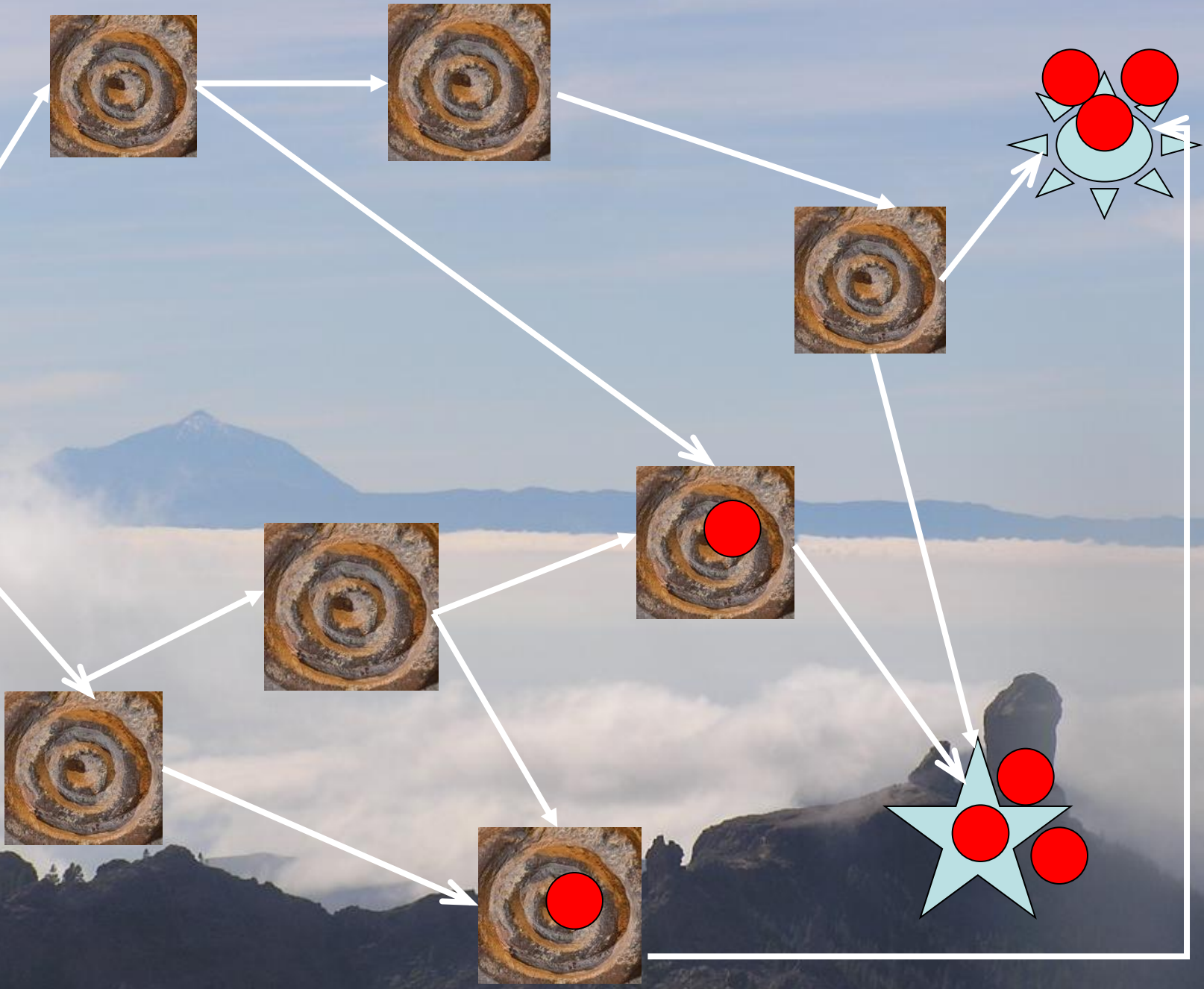
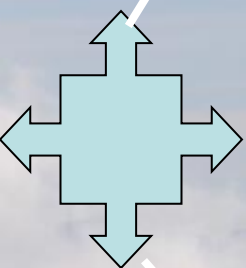
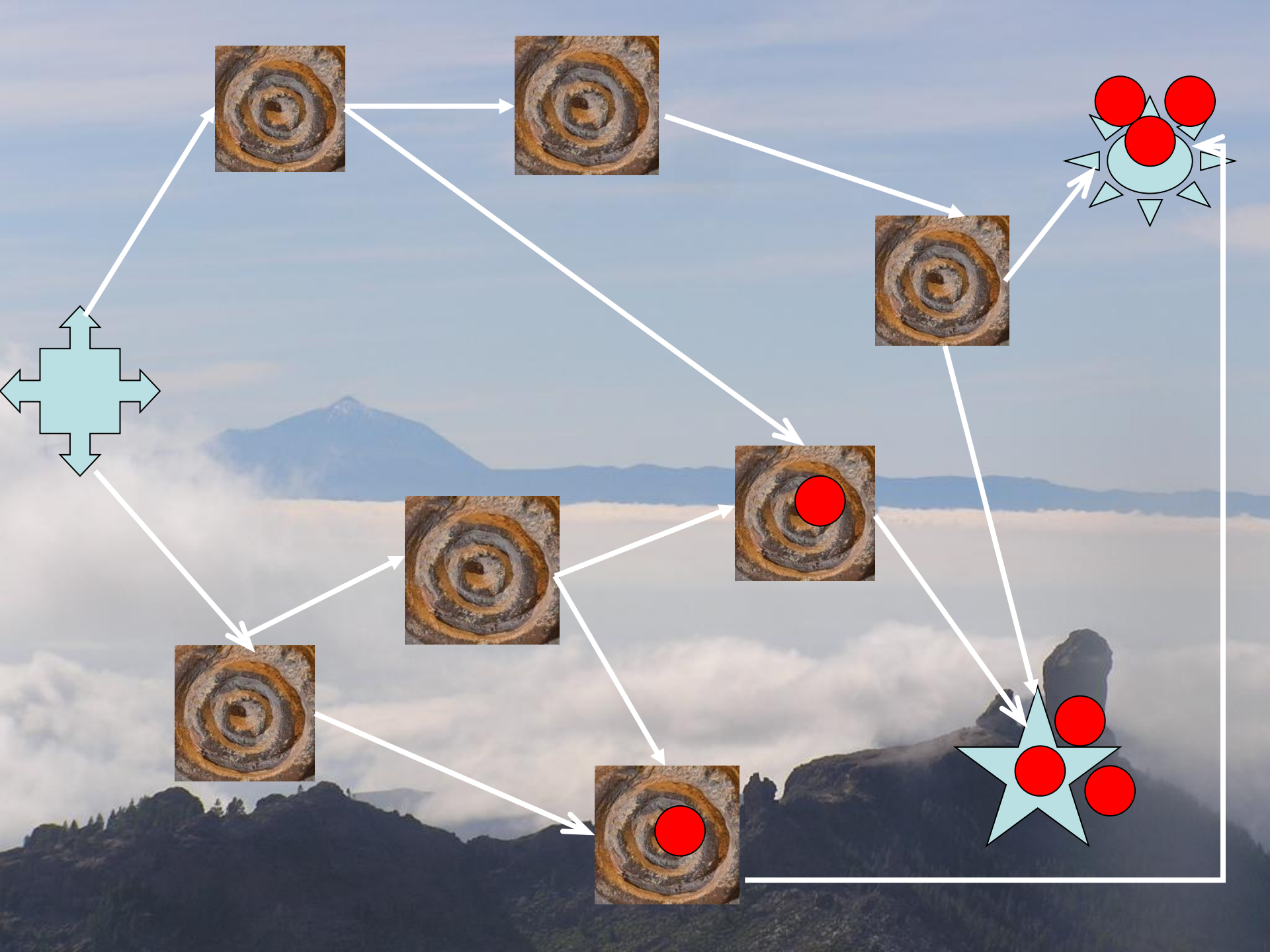


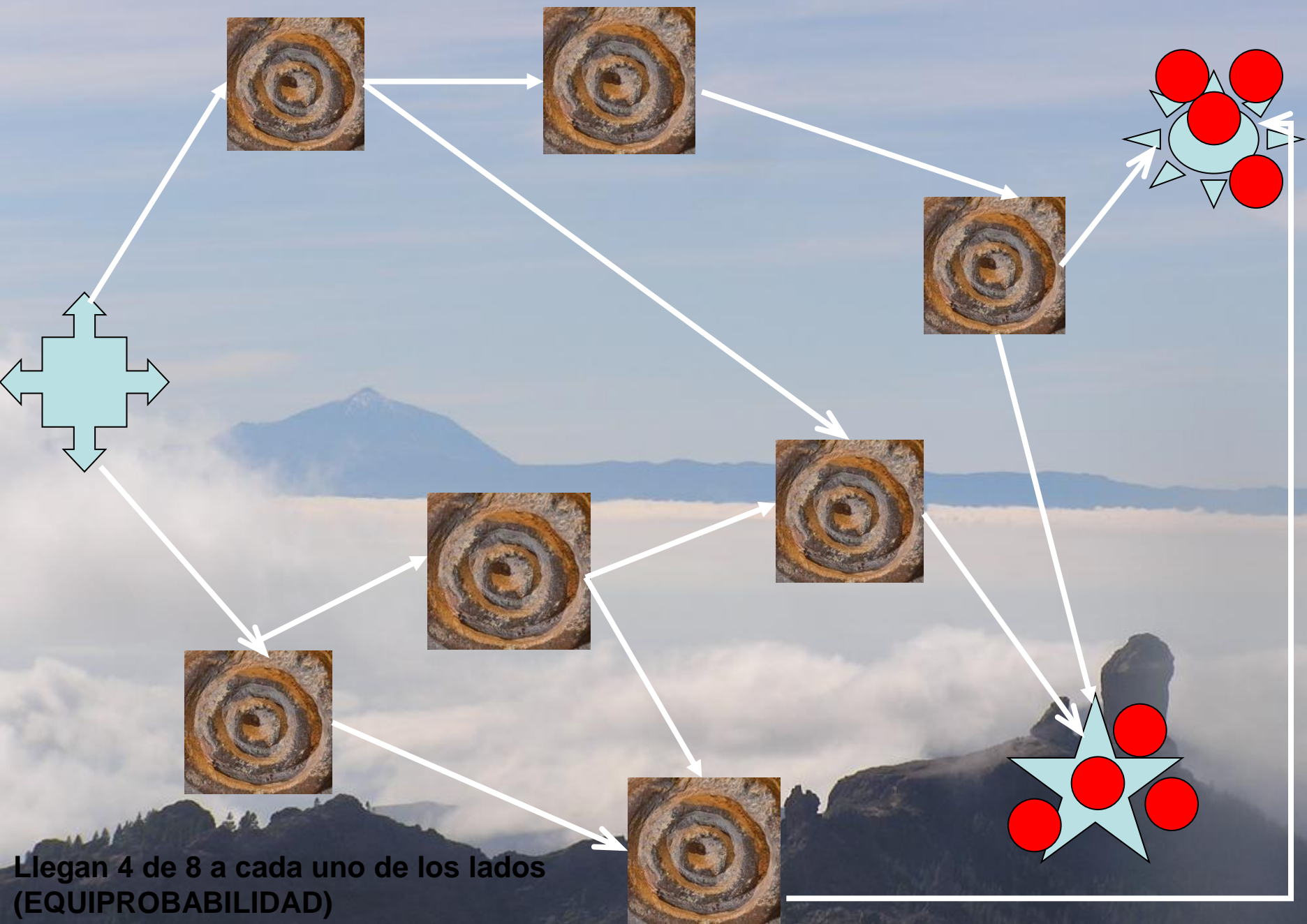






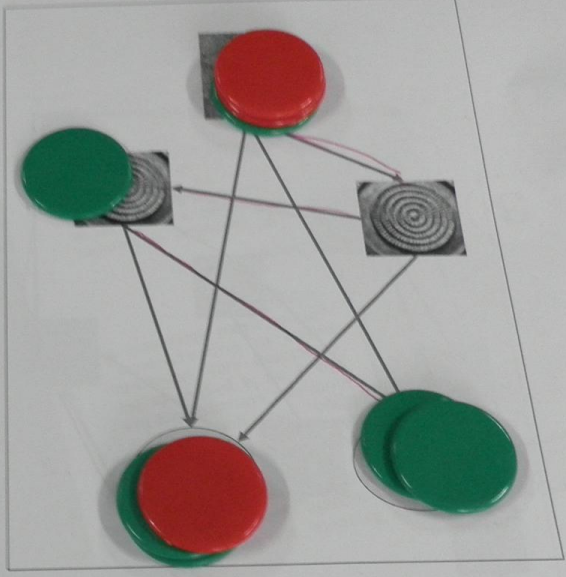
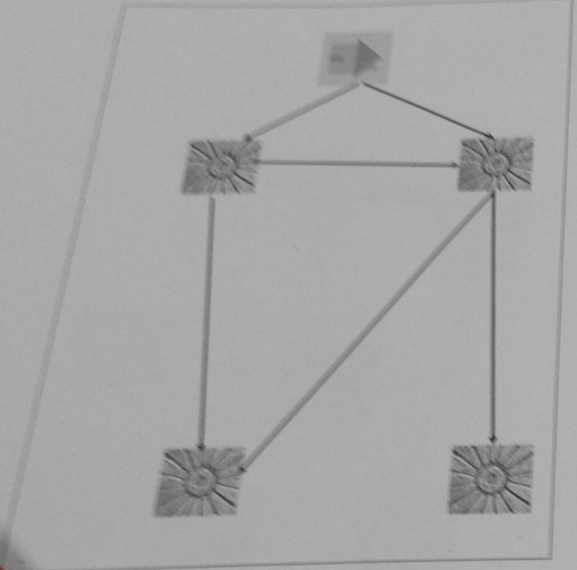
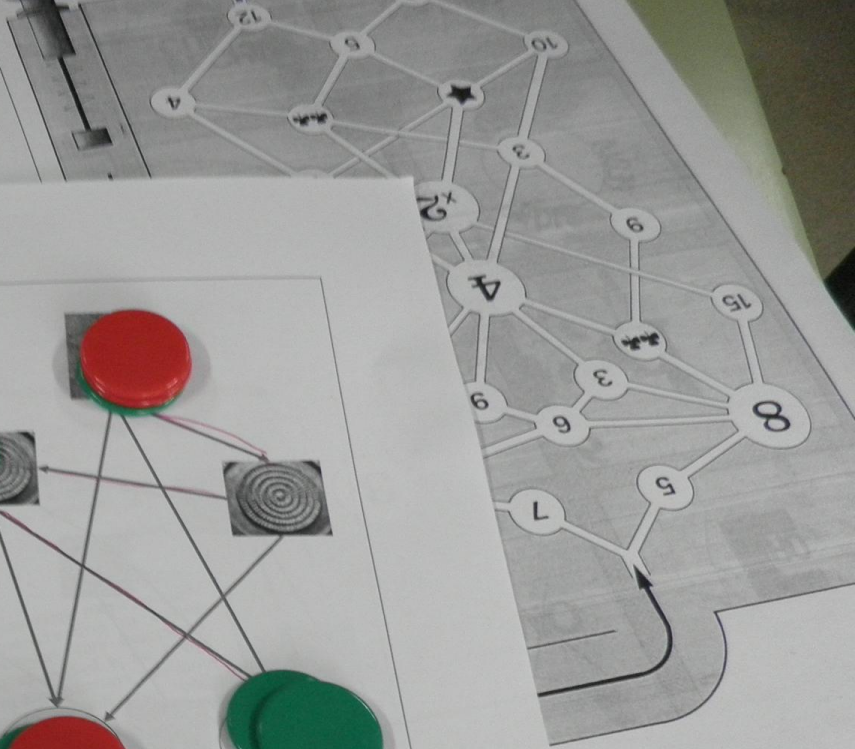




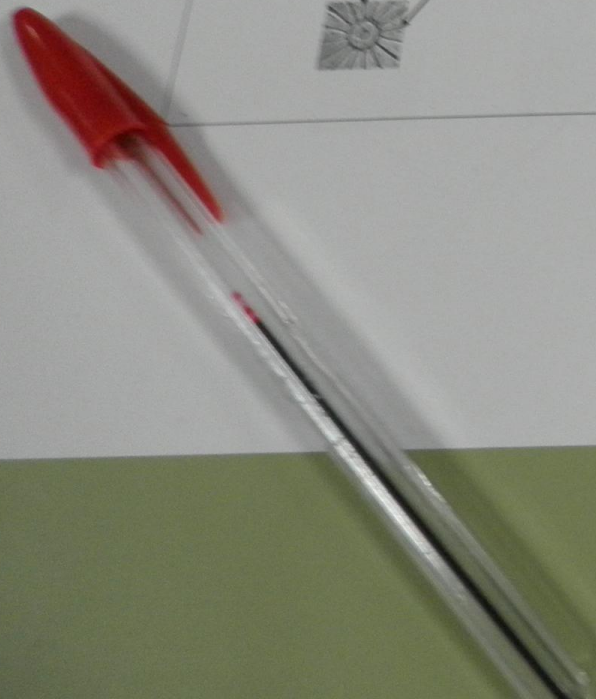


Llegan 4 de 8 a cada uno de los lados  
(EQUIPROBABILIDAD)

Si da usted  
la vuelta a  
la página, po-  
drá leer esta  
frase más co-  
modamente.



Usted conocia  
bola, el que sale  
pin ball. Ese hor



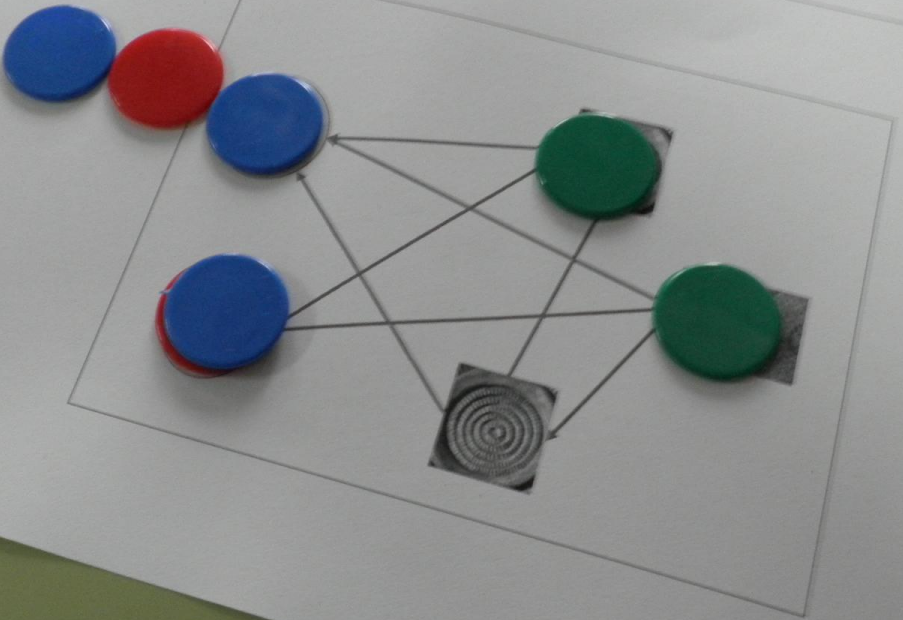
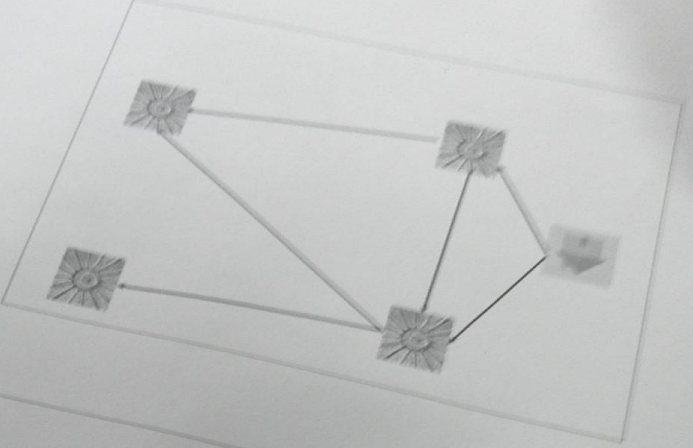
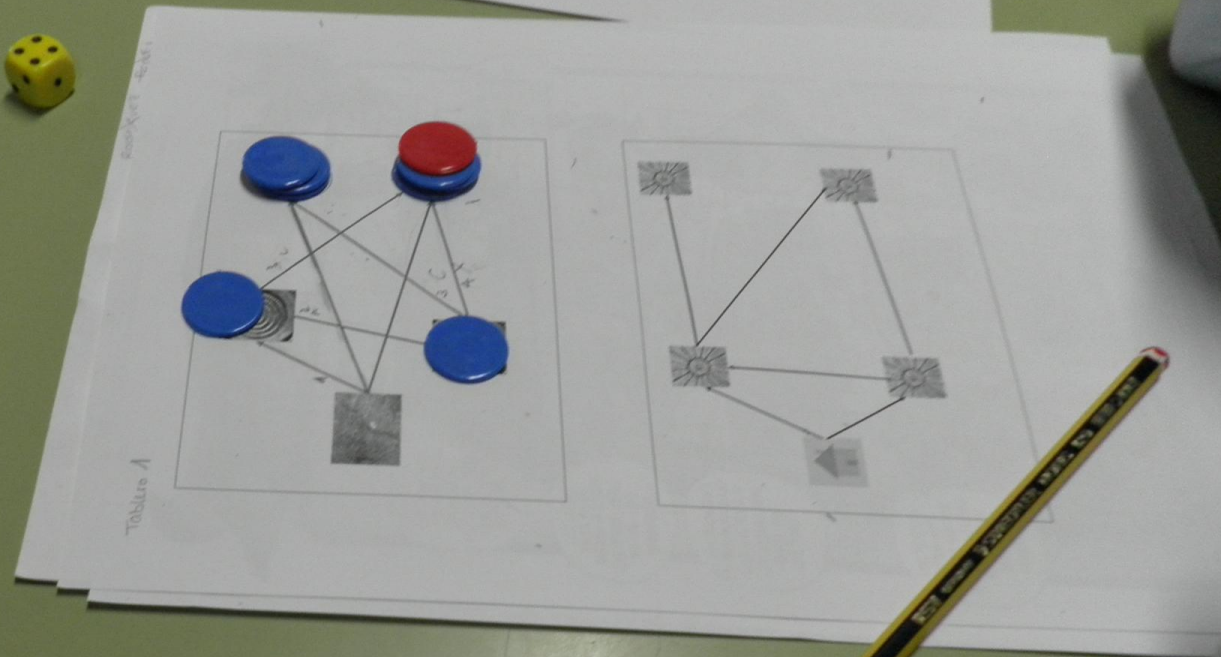


Table A

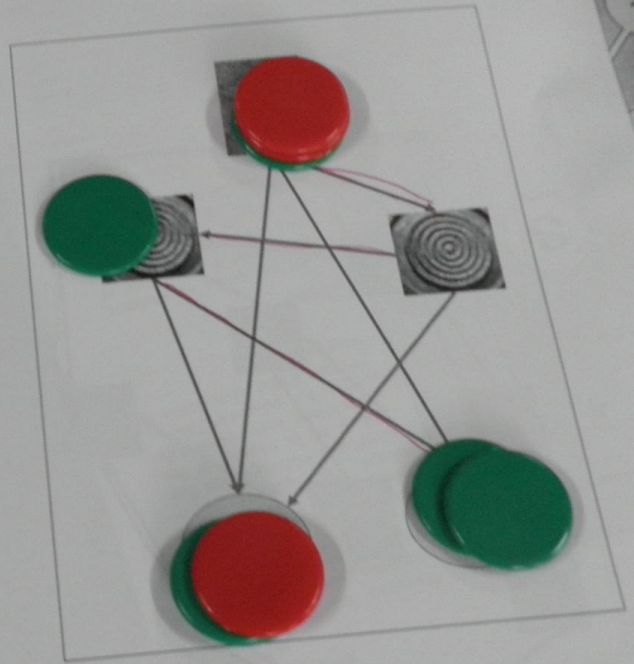
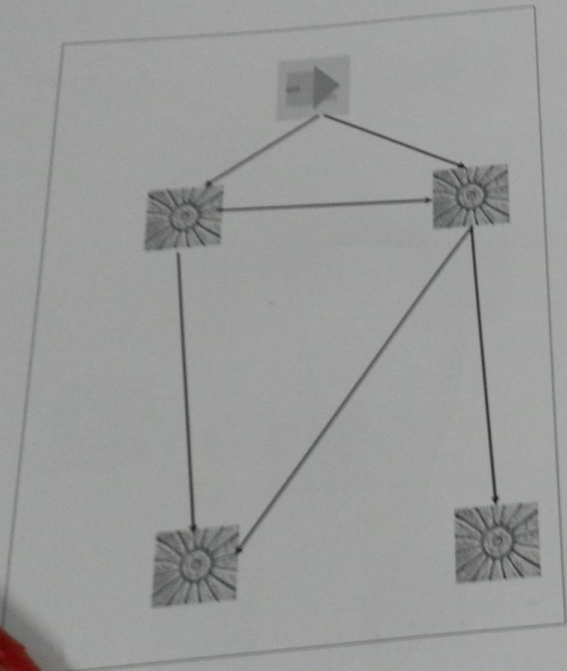
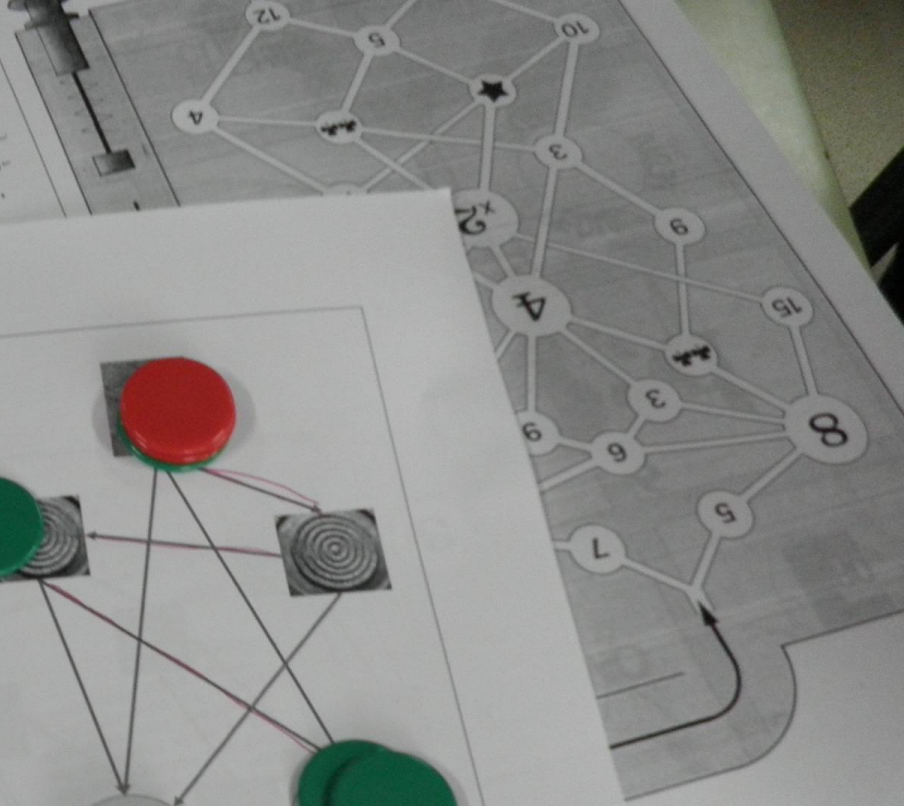
$1c - c2 = 50$   
 $2c - c1 = 20$   
 $3c - c3 = 30$   
 $4c - c4 = 40$   
 $5c - c5 = 50$







Si da usted  
la vuelta a  
la página, po-  
drá leer esta  
frase más co-  
modamente.



¿usted conocia  
bola, el que sale  
pin ball. Ese hoy





# REGLA DE LAPLACE

Una vez realizado el juego se le da la definición clásica de probabilidad para que hallen los resultados de cada uno de los tableros, pero sólo contando las fichas que han llegado a cada sitio.



**“La probabilidad de un suceso elemental es igual al cociente entre el número de casos favorables a ese suceso y el número de casos posibles”**



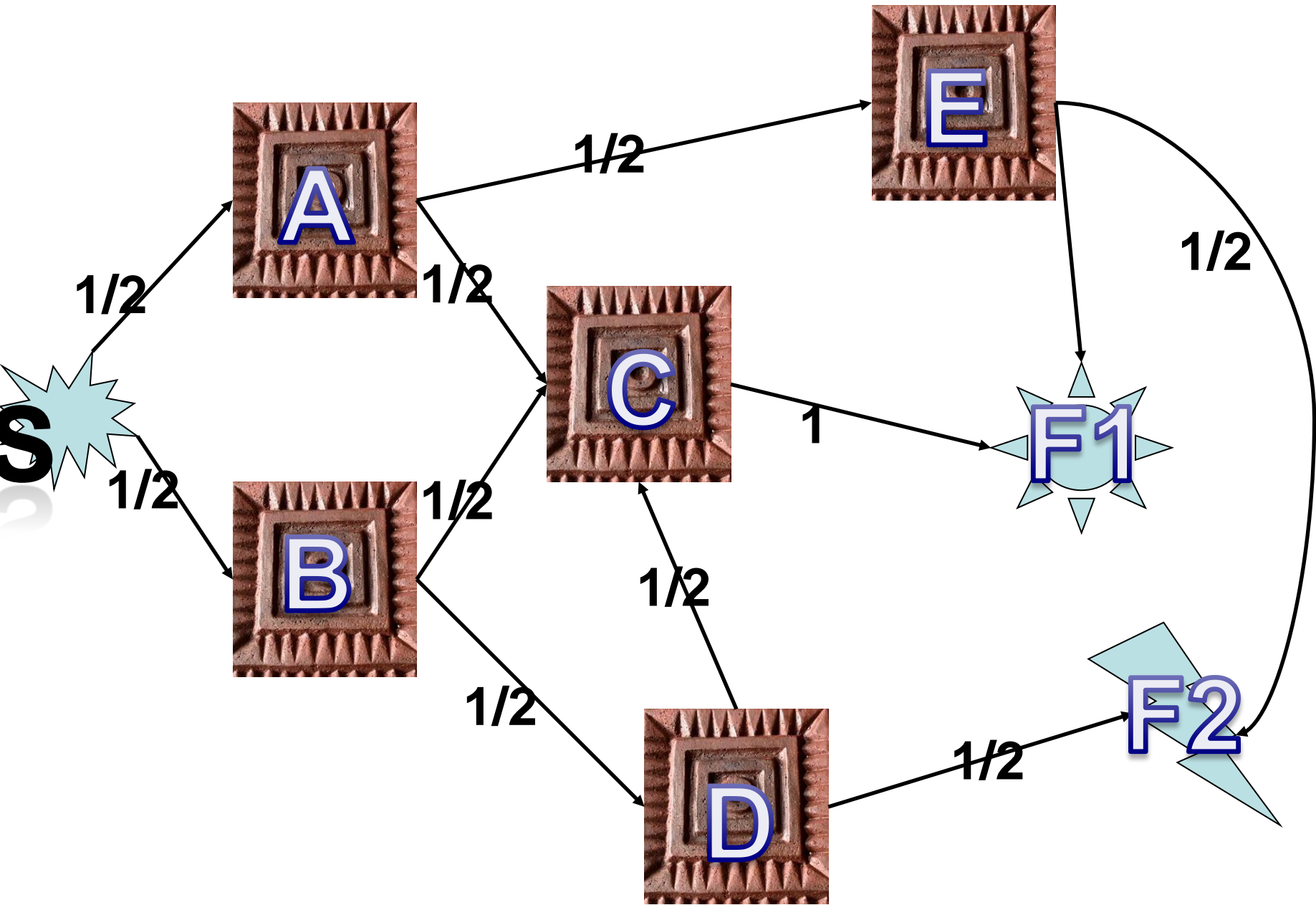
# Diagrama de árbol

Es una representación gráfica que muestra el desglose progresivo de los factores que pueden contribuir a un efecto u objetivo determinado.

El esquema del diagrama de árbol surge como una simplificación natural de un tablero y además le podemos asignar a cada una de las ramas sus respectivas probabilidades.



**Vamos a calcular las probabilidades  
a partir del diagrama de árbol**



Para ello empezamos escribiendo los caminos que nos llevan a cada una de las dos posiciones finales:

**Caminos que llegan al sol:**

***SAESol***

***SACSol***

***SBCSol***

***SBDCSol***

**Caminos que llegan al rayo:**

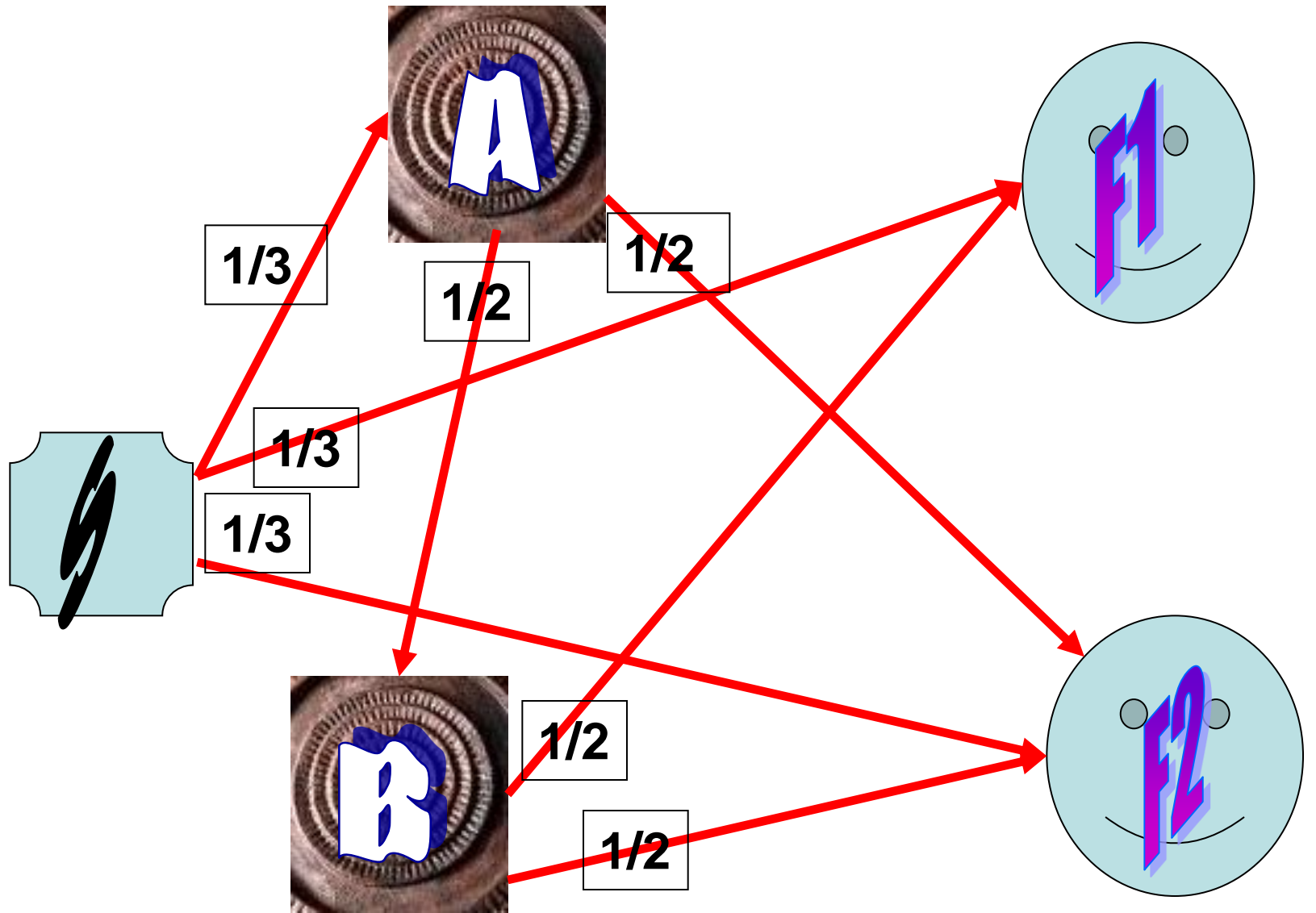
**SAERayo**

**SBDRayo**

Teniendo en cuenta las probabilidades de cada uno de los caminos:

$$\begin{aligned} P(Sol) &= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 1 = \\ &= \frac{1}{8} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} = \frac{6}{8} = \frac{3}{4} \end{aligned}$$

$$P(Rayo) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$$



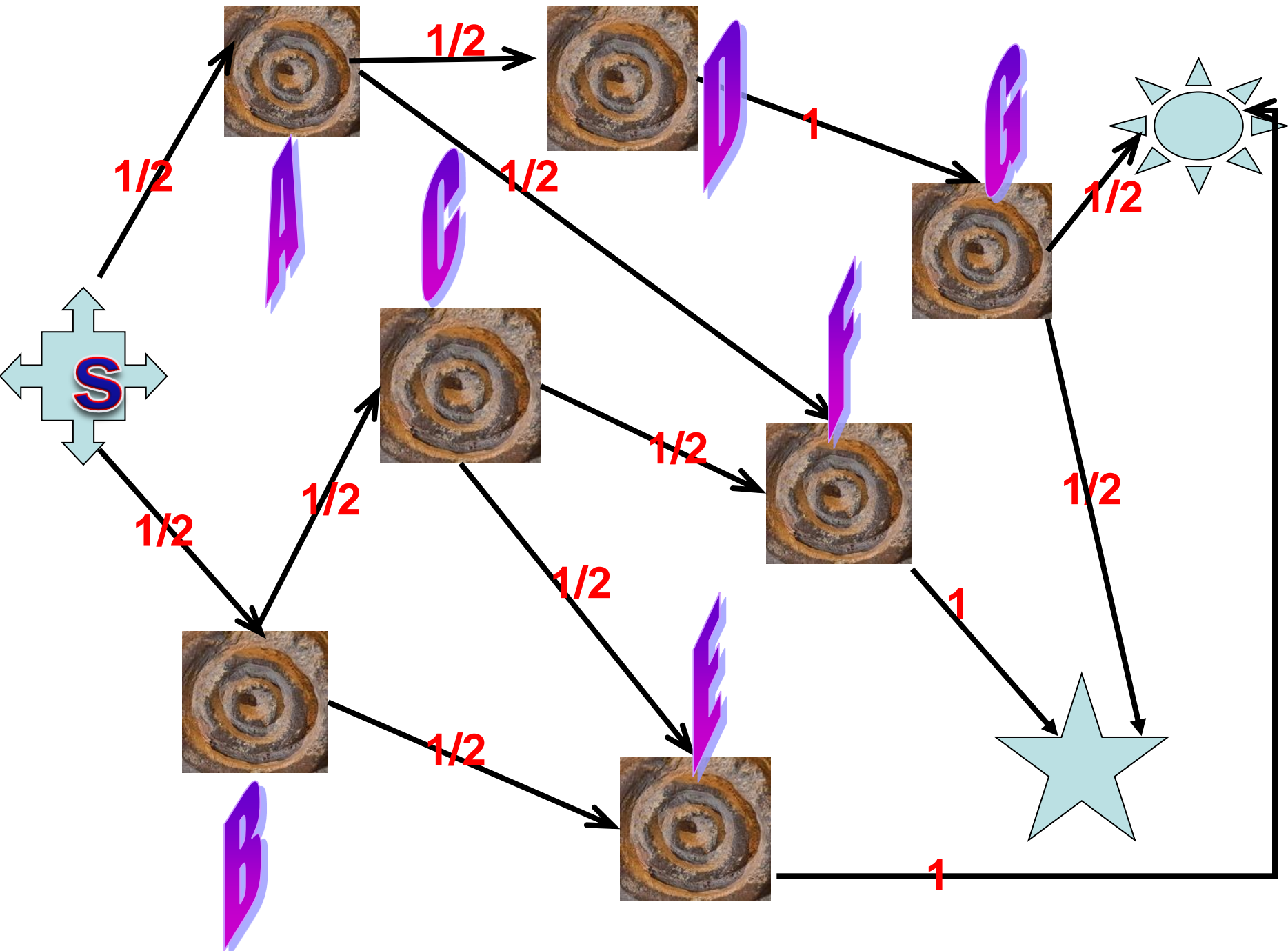
Para ello empezamos escribiendo los caminos que nos llevan a cada una de las dos posiciones finales:

- **Caminos para llegar a F1:**
- **SF1**
- **SABF1**
  
- **Caminos para llegar a F2:**
- **SF2**
- **SAF2**
- **SABF2**

Teniendo en cuenta las probabilidades de cada uno de los caminos:

$$P(F1) = \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{3} + \frac{1}{12} = \frac{5}{12}$$

$$P(F2) = \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{3} + \frac{1}{6} + \frac{1}{12} = \frac{7}{12}$$



Para ello empezamos escribiendo los caminos que nos llevan a cada una de las dos posiciones finales:

**Caminos al Sol**

**SADGSol**

**SBESol**

**SBCESol**

**Caminos a la estrella**

**SAFEstrella**

**SADGEstrella**

**SBCFEstrella**

Teniendo en cuenta las probabilidades de cada uno de los caminos:

$$P(Sol) = \frac{1}{2} \cdot \frac{1}{2} \cdot 1 \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 1 = \frac{1}{8} + \frac{1}{4} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

$$P(Est) = \frac{1}{2} \cdot \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 1 = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

# BIBLIOGRAFÍA:

- ENGEL, A. (1975) “The Probabilistic Abacus” Educational Studies in Mathematics, 6.
- García Cruz, Juan Antonio y Espinel Febles, Candelaria (1994) “El azar en el curriculum” Revista Interuniversitaria de Formación del Profesorado, nº 21